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Cooperation emergence in group population with unequal competitions

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Abstract – Groups are the basic unit of organization in our society. It is important to investigate how groups compete and cooperation evolves in the population composed of groups. In this paper, based on the celebrated multi-player public goods game, we propose a general theoretical framework of the stochastic dynamic process to study how inequalities among groups affect the evolution of cooperation in a group population. We find that cooperation can be promoted if enhancement factors of every group are not equivalent and constant, but are dynamic adjusted according to the group reputation which is defined as the fraction of cooperators in the last public goods game. Furthermore, we introduce the inequality in social roles of groups by means of heterogeneous graph and find that, under the influence of dynamic adjustment of enhancement factors, central groups can always own more cooperators compared to peripheral groups during game dynamics. Moreover, a highly heterogeneous group connecting patterns can help cooperators to survive in the population when the enhancement factor is small, but prevents the spread of cooperation more widely even if the enhancement factor increases.

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Introduction. – Altruistic behavior is important to our society. Research based on evolutionary game theory has revealed many mechanisms that explain the evolution of cooperation among selfish individuals [1–4], including network reciprocity [5–8], coevolution of strategies and population structure [9–11], utilizing information from past [12,13], reward and punishment [14–18], reciprocity of individual reputation [19–24] and stochastic games [25,26].

In our social life, the population is always organized into various groups. Individuals' benefits are influenced by the behaviors of other individuals in the same group directly. Public goods game (PGG) is a useful model to depict the interaction among selfish individuals [27]. The evolution of social dilemmas in group population and how cooperation prevails through intragroup and intergroup interactions have been studied in previous works. Taulsen *et al.* [28] studied the model where an individual only donates to another individual who has a similar tag. Antal and Tarnita [29,30] discussed the coevolution of strategies and population structure in the group population. Wang *et al.* [31] indicated that cooperation is promoted if imitation between individuals belonging to different modules is strong and the imitation within the same module is weak. However, these works almost assume that groups have the same productivity and social roles, but do not consider the inequality among groups.

The effects of inequality and dissimilarity among individuals on the evolution of cooperation have been widely investigated [32–38]. These works assume that each individual has different connecting patterns, productivity, social state and so on. Here, we propose a general stochastic dynamic process to study how different kinds of inequalities among groups affect cooperation in the group

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population. In our society, the productivity of groups is always diverse and dynamic, as resources and preferential policies always favor groups with good reputations. Group reputation has been revealed as a key to promote cooperation in many experimental researches [39–41]. In this work, we assume that each group has a reputation which is formulated by the fraction of cooperators in the last public goods game. Groups compete with each other for a larger enhancement factor based on their reputations. We call this mechanism as enhancement factor competitions based on group reputation. By modeling this mechanism into the stochastic dynamic process model proposed in this work, we give a numerical analysis on the dynamics of cooperation in each group under the influence of enhancement factor competitions. Furthermore, we introduce the inequality in social roles of groups by means of heterogeneous graph [35]. Because of the geographical, political and cultural factors, groups can only interact and compete locally, along the social ties. Heterogeneous group connecting patterns reflect the fact that groups have different roles. We investigate how diverse social roles of each group affect the dynamics of cooperation in them and whether the fraction of cooperators in the group population is influenced by the heterogeneity of social roles.

Evolutionary process. – In this work, it is assumed that each individual belongs to one group with n individuals and participates in a public goods game which is established by all individuals in this group. Individuals can choose either cooperation or defection. A cooperator contributes an amount $c = 1$ to the public goods game, while defectors do nothing. The sum of all these contributions is multiplied by an enhancement factor r, reflecting the synergetic effect of cooperation, and then shared equally among all individuals in the same group irrespective of their contributions. The payoffs of cooperators and defectors are $f_C = \gamma ck - c$ and $f_D = \gamma ck$, respectively. Here, $\gamma = r/n$ is the normalized enhancement factor and k is the number of cooperators in one group.

Individuals are initialized to be cooperators or defectors equally. After playing one round of the public goods game, some individuals will seek for more successful strategies. In every group, a random individual i changes his strategy by mutation and imitation processes. With a mutation rate μ, the individual i adopts the other strategy. With probability $1 - \mu$, the individual i updates his strategy through imitating the behavior of another successful individual j who is picked randomly from his own or other groups. On the basis of the payoff-oriented preferential learning mechanism, the individual i learns j's strategy with probability [6]

$$W(s_i \to s_j) = \frac{1}{1 + \exp[\beta(f_i - f_j)]}, \tag{1}$$

where s_i and s_j are the strategies of individual i and j, respectively. The parameter $\beta \in [0, \infty)$ denotes the amplitude of noise and the intensity of selection ($\beta \to 0$ represents neural drift and $\beta \to +\infty$ represents determinate imitation dynamics) [42,43]. We use $\beta = 2$ in this work [17,18,44].

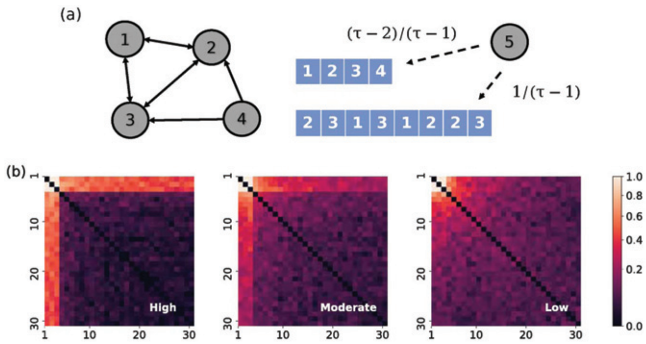

Fig. 1: Heterogeneous group connecting patterns in the group population. Panel (a) illustrates the generation process of heterogeneous network. With probability $(\tau - 2)/(\tau - 1)$, a new node will be connected to a random existing node. With probability $1/(\tau - 1)$, the connected node is picked randomly from the ending nodes list. Panel (b) shows the stochastic adjacent matrices with high, moderate and low heterogeneity. The elements of these matrices indicate the probability of pairs of vertices being adjacent. The diagonal elements are set as 0.

Considering enhancement factor competitions based on group reputation, the normalized enhancement factors of every group change according to their group reputations. The normalized enhancement factors of every group are all initialized as $\bar{\gamma}$ at first. With the assumption that there is a number of m groups, $\bar{\gamma}m$ represents the whole resources and supplies all groups can get to increase their productivity. The normalized enhancement factor of group u at time t is defined as

$$\gamma_u(t) = \sum_{\substack{v=1 \\ v \neq u}}^{m} \frac{\bar{\gamma}m}{d} \frac{\frac{k_u(t-1)}{n} + \epsilon}{\frac{k_u(t-1)}{n} + \frac{k_v(t-1)}{n} + 2\epsilon}, \tag{2}$$

where d is the number of pairs of groups and every enhancement factor competition happens between two groups. That is to say, $\bar{\gamma}m$ is divided into d parts and a pair of groups need to compete with each other for one part, $\bar{\gamma}m/d$. If groups are all connected, d equals $m(m-1)/2$. The reputation of group u at time t is defined as the fraction of cooperators belonging to group u at time $t-1$, $k_u(t-1)/n$. The group who has a higher percentage of cooperators at time $t-1$ can get a bigger part of $\bar{\gamma}m/d$ at time t. ϵ is an approaching zero positive value to avoid division by zero and it is set as 0.001 in this work. The payoffs of cooperators and defectors in group u with k_u cooperators at time t become $f_{u,C}(t) = \gamma_u(t)ck_u - c$ and $f_{u,D}(t) = \gamma_u(t)ck_u$, respectively.

The inequality in social roles of groups will be considered through a perspective of heterogeneous network, where nodes are the groups and links represent the social ties between groups. The heterogeneous network is generated as follows [45–47]: without loss of generality,

$$T_u^+(k_u) = \sum_{\substack{v=1\\v\neq u}}^{m}\sum_{k_v=0}^{n}\frac{1}{m}\frac{n-k_u}{n}\frac{k_v}{n}x_{v,k_v}[1+e^{-\beta[f_{u,D}(k_u)-f_{v,C}(k_v)]}] + \frac{1}{m}\frac{n-k_u}{n}\frac{k_u}{n}[1+e^{-\beta[f_{u,D}(k_u)-f_{u,C}(k_u)]}],$$

$$T_u^-(k_u) = \sum_{\substack{v=1\\v\neq u}}^{m}\sum_{k_v=0}^{n}\frac{1}{m}\frac{k_u}{n}\frac{n-k_v}{n}x_{v,k_v}[1+e^{-\beta[f_{u,C}(k_u)-f_{v,D}(k_v)]}] + \frac{1}{m}\frac{k_u}{n}\frac{n-k_u}{n}[1+e^{-\beta[f_{u,C}(k_u)-f_{u,D}(k_u)]}], \quad (4)$$

$$\gamma_{u,k_u}(t) = \sum_{\hat{k}_u=k_u-1}^{k_u+1}\sum_{\substack{v=1\\v\neq u}}^{m}\sum_{k_v=0}^{n} P_u(\hat{k}_u|k_u)x_{v,k_v}(t-1)\frac{\bar{\gamma}m}{d}\frac{\frac{\hat{k}_u}{n}+\epsilon}{\frac{\hat{k}_u}{n}+\frac{k_v}{n}+2\epsilon}, \quad (6)$$

the network generation process starts from a fully connected network of nodes with index 1, 2, 3 and builds an ending nodes list which is composed of nodes staying at the end of every edge. Then one new node, following the index ordering, is connected to two existing nodes in the network at each time. With probability $1/(\tau-1)$, the node to be connected is picked from the ending nodes list. Otherwise, the node is picked from all of the existing nodes randomly, which is shown in fig. 1(a). The connected nodes are added to the ending nodes list. Parameter τ controls the level of heterogeneity and $\tau \in (2,\infty)$. As τ increases, the group connecting patterns become less heterogeneous. In this work, we consider three levels of heterogeneity: high ($\tau = 2.2$), moderate ($\tau = 3.0$) and low ($\tau = 10.0$). When heterogeneity is high, most nodes connect to the first three nodes and the remaining ones are rarely connected with each other. For moderate and low heterogeneity, more and more connections appear between nodes except for the first three nodes.

In the population where groups are locally connected, individuals in one group can only contact individuals in their neighboring groups, $v \in \Omega(u)$, where $\Omega(u)$ represents the set of neighboring groups belonging to group u. The imitation process can only take place between individuals in the same or connected groups. The enhancement factor competitions also just happen in the neighborhood and eq. (2) turns out to be

$$\gamma_u(t) = \sum_{v\in\Omega(u)}\frac{\bar{\gamma}m}{d}\frac{\frac{k_u(t-1)}{n}+\epsilon}{\frac{k_u(t-1)}{n}+\frac{k_v(t-1)}{n}+2\epsilon}, \quad (3)$$

where d represents the number of pairs of groups that compete with each other. As the groups are locally connected, parameter d equals the number of links in the group connecting patterns.

Stochastic dynamic process. – For understanding the evolutionary process described above more precisely and clearly, we propose a stochastic dynamic process to represent the evolutionary process. The vector with $n+1$ dimension, $\boldsymbol{x}_u = [x_{u,0}, x_{u,1}, \ldots, x_{u,k}, \ldots, x_{u,n}]$, is used to demonstrate the distribution of the number of cooperators in group u. $x_{u,k}$ represents the probability that there are k cooperators in group u. If each individual in group u is initialized to be cooperative with probability π_u, $x_{u,k} = C_n^k\pi_u^k(1-\pi_u)^{n-k}$ at first. In this work, π_u is set as 0.5. The fraction of cooperators in group u and the whole population can be calculated as $\bar{x}_u = \sum_{k=0}^{n} kx_{u,k}$ and $\bar{x} = \frac{1}{m}\sum_{u=1}^{m}\bar{x}_u$, respectively. Based on the strategy update rule and the pair-wise comparison process defined above, the probability that the number of cooperators of group u increases or decreases by one can be written as [43,48,49]

see eq. (4) *above*

where k_u and k_v are the numbers of cooperators in group u and v, respectively. $f_{u,C}(k_u)$ and $f_{u,D}(k_u)$ are the payoffs of cooperators and defectors in group u containing k_u cooperators. Considering the mutation rate μ, the transition probabilities become $T^+_{\mu,u}(k_u) = (1-\mu)T^+_u(k_u) + \mu(n-k_u)/n$ and $T^-_{\mu,u}(k_u) = (1-\mu)T^-_u(k_u) + \mu k_u/n$. For group u, a transition matrix $S_u = [p_{ij}]$ is defined as $p_{k_u,k_u\pm1} = T^{\pm}_{\mu,u}(k_u)$ and $p_{k_u,k_u} = 1 - p_{k_u,k_u+1} - p_{k_u,k_u-1}$. The updating process of $\boldsymbol{x}_u$ can be described as

$$\boldsymbol{x}_u(t+1) = \boldsymbol{x}_u(t)\cdot S_u(t). \quad (5)$$

With enhancement factor competitions based on group reputation, the enhancement factor of group u with k_u cooperators in the stochastic dynamic process at time t can be written as

see eq. (6) *above*

where $P_u(\hat{k}_u|k_u)$ is the probability that, in group u, there are $\hat{k}_u$ cooperators at time $t-1$, if there are k_u ones at time t. $P_u(\hat{k}_u|k_u)$ can be obtained from

$$P_u(\hat{k}_u|k_u) = \frac{p_{\hat{k}_u,k_u}(t-1)x_{u,\hat{k}_u}(t-1)}{\sum_{\hat{k}_u=k_u-1}^{k_u+1} p_{\hat{k}_u,k_u}(t-1)x_{u,\hat{k}_u}(t-1)}. \quad (7)$$

The payoffs of cooperators and defectors in group u with k_u cooperators at time t become $f_{u,C}(k_u) = \gamma_{u,k_u}(t)ck_u - c$ and $f_{u,D}(k_u) = \gamma_{u,k_u}(t)ck_u$, respectively.

In order to compute the stochastic dynamic process when groups are connected locally, we use a stochastic

$$T_u^+(k_u)=\sum_{\substack{v=1\\v\neq u}}^{m}\sum_{k_v=0}^{n}\frac{\bar{a}_{uv}}{\bar{m}_u}\frac{n-k_u}{n}\frac{k_v}{n}x_{v,k_v}[1+e^{-\beta[f_{u,D}(k_u)-f_{v,C}(k_v)]}]+\frac{1}{\bar{m}_u}\frac{n-k_u}{n}\frac{k_u}{n}[1+e^{-\beta[f_{u,D}(k_u)-f_{u,C}(k_u)]}],$$

$$T_u^-(k_u)=\sum_{\substack{v=1\\v\neq u}}^{m}\sum_{k_v=0}^{n}\frac{\bar{a}_{uv}}{\bar{m}_u}\frac{k_u}{n}\frac{n-k_v}{n}x_{v,k_v}[1+e^{-\beta[f_{u,C}(k_u)-f_{v,D}(k_v)]}]+\frac{1}{\bar{m}_u}\frac{k_u}{n}\frac{n-k_u}{n}[1+e^{-\beta[f_{u,C}(k_u)-f_{u,D}(k_u)]}], \quad (8)$$

$$\gamma_{u,k_u}(t)=\sum_{\hat{k}_u=k_u-1}^{k_u+1}\sum_{\substack{v=1\\v\neq u}}^{m}\sum_{k_v=0}^{n}P_u(\hat{k}_u|k_u)x_{v,k_v}(t-1)\frac{\bar{\gamma}m\bar{a}_{uv}}{\bar{d}}\frac{\frac{\hat{k}_u}{n}+\epsilon}{\frac{\hat{k}_u}{n}+\frac{k_v}{n}+2\epsilon}, \quad (9)$$

adjacent matrix $\bar{A}$ to illustrate the connection states between groups. In the stochastic adjacent matrix $\bar{A}$, parameter $\bar{a}_{uv}\in[0,1]$ means the probability that there is a link between nodes u and v. For generating stochastic adjacent matrix $\bar{A}$, a bunch of networks are built based on the generative model described above and $\bar{a}_{uv}=\frac{1}{H}\sum_{i=1}^{H}a_{uv}^i$, where H is the number of built networks and a_{uv}^i is the element of adjacent matrix A^i of the i-th built network. In this work, $\bar{a}_{uu}=0$ and $H=100$. For each level of heterogeneity, the stochastic adjacent matrices are shown in fig. 1(b).

Considering the stochastic dynamic process where groups are connected locally, we can calculate the parameters $T_u^+(k_u)$ and $T_u^-(k_u)$ based on the stochastic adjacent matrix as

see eq. (8) *above*

where $\bar{m}_u=\sum_{v\in\Omega(u)}\bar{a}_{uv}+1$. The enhancement factor of group u with k_u cooperators at time t can be written as

see eq. (9) *above*

where $\bar{d}=\sum_{u=1}^{m}\sum_{v=1}^{m}\bar{a}_{uv}/2$.

Results. – We first assume that groups are all connected with each other and study how enhancement factor competitions influence the evolution of cooperation. In this work, it is assumed that there are $n=5$ individuals in each group and there are $m=30$ groups totally in the population. The relationship between fraction of cooperators in the group population, $\bar{x}$, and average enhancement factor, $\bar{\gamma}$, is shown in fig. 2. The results of the stochastic dynamic process concide well with the results of the evolutionary process, which means that our proposed stochastic dynamic process model can describe the evolutionary process in group population well. Without considering enhancement factor competitions, cooperators begin to survive in the population when $\bar{\gamma}$ equals about 0.8 and occupy the whole population when $\bar{\gamma}$ increases to 1.2. However, enhancement factor competitions can decrease these values to 0.4 and 0.8, respectively. That is to say, with the help of enhancement factor competitions, cooperators can survive and prevail in the group population with a relatively small average enhancement factor.

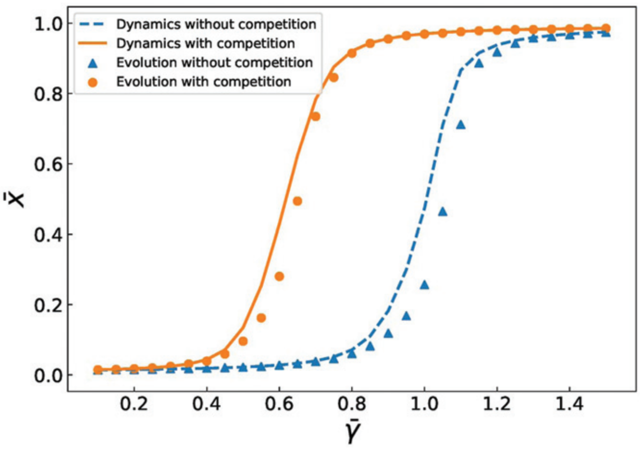

Fig. 2: Fraction of cooperators in the population, $\bar{x}$, as a function of average enhancement factor, $\bar{\gamma}$. The results demonstrated by lines are from the stochastic dynamic process and the results shown by dots are from the evolutionary process. These two kinds of results generated by the corresponding processes are found to be in good agreement. Blue dashed lines and triangles represent the case not considering enhancement factor competitions, while orange solid lines and circles illustrate the results considering enhancement factor competitions. Both the evolution and dynamic process last 1000 time steps. The fraction of cooperators in the group population is obtained by averaging the last 200 time steps.

Furthermore, we investigate how enhancement factor competitions based on group reputation affect the dynamics of cooperation in each group. Figure 3 illustrates how the probabilities that there are k cooperators in one group change with time. Figures 4(a) and (b) demonstrate the direction of dynamics for different k and the exact transition matrices are shown in figs. 4(c) and (d).

In a situation without enhancement factor competitions, fig. 3(a) shows that the probability of $k=0$ can increase to 0.82 at steady state. The probabilities of k equalling 1 and 2 take a few time rising and then decline to about 0.2 and 0, respectively. Meanwhile, the probabilities of $k=3,4,5$ drop to 0. It can also be found in fig. 4(a) that $\alpha(k)$ finally becomes bigger than 1 no matter what k is.

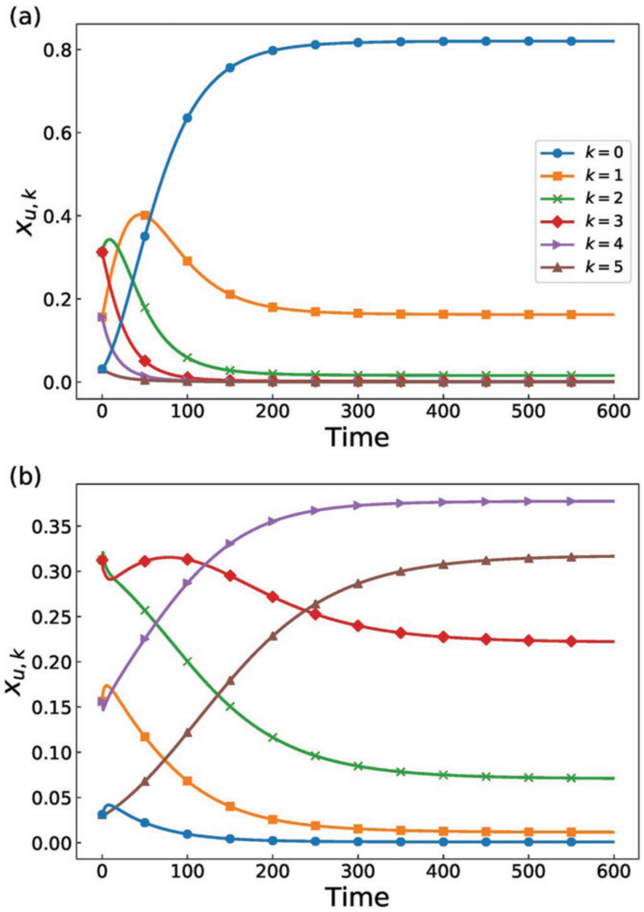

Fig. 3: The dynamics of probabilities that there are k cooperators in one group with time. $x_{u,k}$ represents the probability of k cooperators being in group u. It is the k-th element in $\boldsymbol{x}_u$. The updating process of $\boldsymbol{x}_u$ is described in eq. (5). Panel (a) is for results not considering enhancement factor competitions. Panel (b) is for results considering them. Different line markers are for different k. $\bar{\gamma}$ is set as 0.7 in these figures.

One special case is that, for $k = 1$, $\alpha(k)$ is smaller than 1 at the beginning of dynamics. Indeed, for $k = 0, 1$, the value $T^+_{\mu,u}(k)$ is relatively high at first and then decreases as time proceeds, which is shown in fig. 4(c). The reason is that the payoffs of defectors in groups with $k = 0, 1$ are smaller than cooperators in groups with $k = 3, 4, 5$. However, the equaling enhancement factors in every public goods game cannot support cooperators in resisting the invasion of defectors in groups with the same or similar k.

With enhancement factor competitions based on group reputation, the dynamics of $x_{u,k}$ are quite different in fig. 3(b) compared to fig. 3(a). The probabilities of k equaling 4 and 5 become high in the steady distribution of k, rising to about 0.38 and 0.32, respectively. Meanwhile, the probabilities of k equaling 0 and 1 approach 0 and the probabilities of k equaling 2 and 3 decrease to 0.07 and 0.22, respectively. For $k = 4, 5$, a large number of cooperators can give groups a competitive advantage in the enhancement factor competitions and bring a larger γ to the next public goods games established in them. As there is only one individual in a group to update strategy, there will be 3, 4 or 5 cooperators in these public goods games. Correspondingly, the enhancement factors of public goods games with $k = 0, 1, 2$ are relatively smaller. In fig. 4(b), we can find that all $\alpha(k)$ decrease. For $k = 1$, the ratio $\alpha(k)$ keeps smaller than 1. For $k = 2, 3$, there exists a transition process from $\alpha(k) > 1$ to $\alpha(k) < 1$. The value $T^+_{\mu,u}(k)$ keeps increasing with time for $k = 0, 1, 2, 3$, as shown in fig. 4(d). However, we can also noticed that enhancement factor competitions cannot wipe out the defectors of the group. For $\bar{\gamma} = 0.7$, the probability of $k = 4$ is higher than the probability of $k = 5$ and $\alpha(k)$ is bigger than 1 for $k = 4$. The increased enhancement factor in one group also increases the payoffs of the defectors in this group. Thus, the payoffs of defectors in groups with a relatively high percentage of cooperators can still exceed the payoffs of cooperators in other groups. The defective behavior of such defectors can spread in their own and other groups. This combined effect of enhancement factor competition mechanism and inherit profit of defective behavior results in the steady distribution of number of cooperators in the group.

Next, we consider the situation in which groups are connected locally and have different social roles in the group population. We find that different levels of heterogeneity of group connecting patterns affect the final fraction of cooperators significantly under the influence of enhancement factor competitions. Figure 5 illustrates the fraction of cooperators in the group population, $\bar{x}$, as a function of average enhancement factor, $\bar{\gamma}$, with different levels of heterogeneity. If enhancement factor competitions do not take place, $\bar{x}$ is not influenced by the level of heterogeneity. However, when enhancement factor competitions are taken into active, that is another story. If heterogeneity is high, it can be found that $\bar{x}$ can reach a relatively high value at a relatively small $\bar{\gamma}$. But when $\bar{\gamma}$ rises, $\bar{x}$ cannot increase further. However, for group connecting patterns being less heterogeneous, $\bar{x}$ can reach a high value, almost dominating the whole population, when $\bar{\gamma}$ is large enough. But it comes with a limitation that cooperators disappear in a small average enhancement factor environment. That is to say, along with enhancement factor competitions, a high level of heterogeneity can help cooperators to survive in the population when the average enhancement factor is small, but prevents the cooperative behavior from spreading more widely when the average enhancement factor rises.

Moreover, we investigate how diverse social roles influence the dynamics of cooperation in different groups and whether there is a reciprocity between heterogeneous group connecting patterns and enhancement factor competitions. The final fractions of cooperators in every group are distinct and emerge in a descending order by their social roles. When heterogeneity is high, central groups have much more neighboring groups compared to others. They attend more competitions and accumulate larger

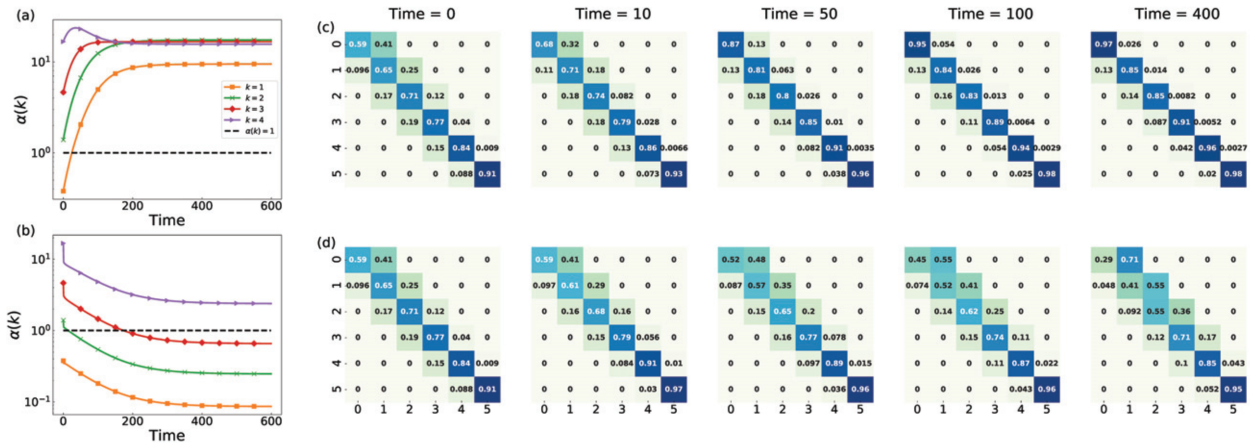

Fig. 4: Analysis of state transitions in the stochastic dynamic process in terms of eq. (4). The upper figures are for the results without enhancement factor competitions. The bottom figures are for the results with them. Panels (a) and (b) show the change of $\alpha(k) = T^{-}_{\mu,u}(k)/T^{+}_{\mu,u}(k)$ with time when $k = 1, 2, 3, 4$ ($T^{-}_{\mu,u}(k) = 0$ if $k = 0$ and $T^{+}_{\mu,u}(k) = 0$ if $k = 5$). The ratio $\alpha(k)$ is used to show the direction of dynamics. $\alpha(k) > 1$ means that the number of cooperators is inclined to increase by one on the condition that there are k cooperators in the group and $\alpha(k) < 1$ is for the decreasing one. Panels (c) and (d) represent the transition matrices of one group at different time. $\bar{\gamma}$ is set as 0.7 in these figures.

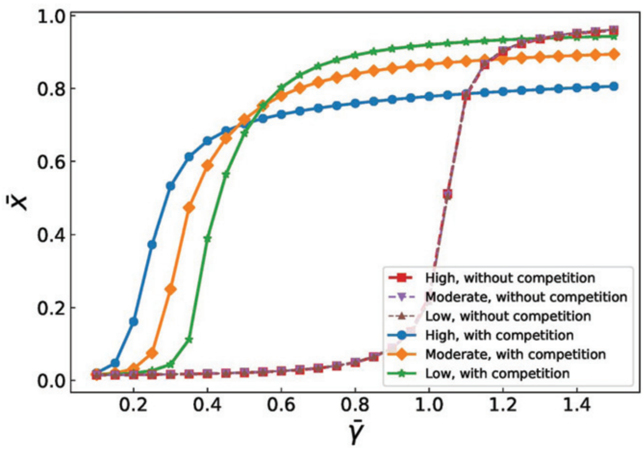

Fig. 5: Fraction of cooperators in the population, $\bar{x}$, as a function of average enhancement factor, $\bar{\gamma}$, in heterogeneous group connecting patterns. Dashed lines with square, down-pointing triangle and up-pointing triangle demonstrate the results not considering enhancement factor competitions with high, moderate and low heterogeneity. Solid lines with circle, diamond and star markers show the results obtained by considering enhancement factor competitions with high, moderate and low heterogeneity.

enhancement factors. Because of the larger enhancement factor, cooperators in central groups could get higher payoffs than defectors in peripheral groups. Thus, they can resist the invasion of defectors from other groups and promote cooperation to them. If the heterogeneity of group connecting patterns becomes low and the average enhancement factor is small, the enhancement factors gathered in central groups are not large enough to support cooperators in surviving in them and the fractions of cooperators in central and peripheral groups decrease with time together. However, when the average enhancement factor becomes large enough, a low heterogeneity group connecting patterns can help peripheral groups to get larger enhancement factors due to the fact that they are able to engage in more competitions. This avoids the redundant accumulation of enhancement factors in central groups and helps cooperation to prevail more widely.

Conclusions and discussions. – In this work, we propose a general stochastic dynamic process model to numerically study the influence of enhancement factor competitions and inequalities among groups on the evolution of cooperation in the group population. The enhancement factor competition mechanism relates the payoffs of an individual with the behaviors of individuals in other groups. It can help an individual who lives in a more cooperative group to get more benefits in the following public goods game, which results in the promotion of cooperation. Moreover, the diversity in social roles of groups can cause the inequality in the final fractions of cooperators of each group. Under the influence of enhancement factor competitions, central groups can seize larger enhancement factors and support more cooperators in surviving in them compared to peripheral groups. The accumulation of enhancement factors in central groups leads to the phenomenon that cooperators can survive and take a relatively big part of the population though the enhancement factor is small, but cannot prevail further and dominate the whole population even if the enhancement factor rises.

However, the enhancement factor competition mehcanism cannot wipe out defectors. Defectors in the group with a relatively high percentage of cooperators can survive and spread their defective behaviors. How to design mechanisms to wipe out such defectors is an interesting topic and we believe that our model and results can give a good foundation. Moreover, this work assumes that an individual only participates in one public goods game. There is also a situation in which individuals attend multiple public goods games simultaneously, which can be captured by evolutionary public goods game in structured population. We think that the enhancement factor competition mechanism can help cooperators to form clusters more easily in the structured population and promote the evolution of cooperation. It is interesting to explore how the enhancement factor competition mechanism works in various kinds of structured population and whether it can promote cooperation further with the coevolution mechanism of population structure.

* * *

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