Chapter 9
Spreading on Complex Networks

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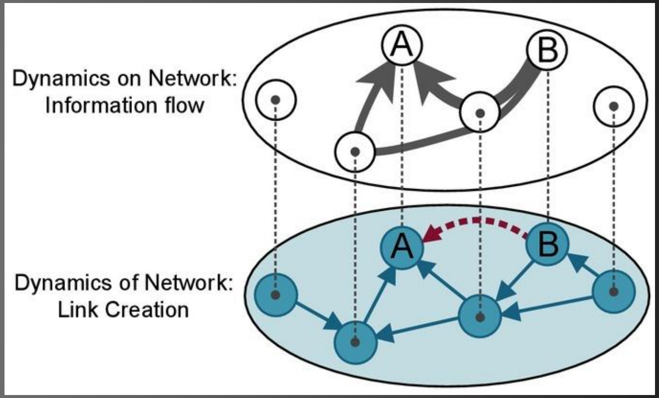
Network Structure & Node Dynamics

Every node has an evolving state:

- Spreading：Infected, Susceptible, Recovered
- Game：Cooperation, Defection
- Mobile Agents：Position, Velocity



Structure-dependent problems:

- Spreading: How many nodes will be infected?
- Game: When an individual choose to cooperate?
- Mobile Agents: How to achieve rendezvous?

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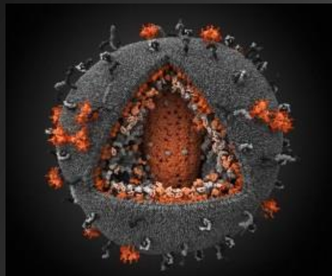

Virus

Rumor

Spreading on Social Networks

Fashion

Opinion

Behavior

Belief

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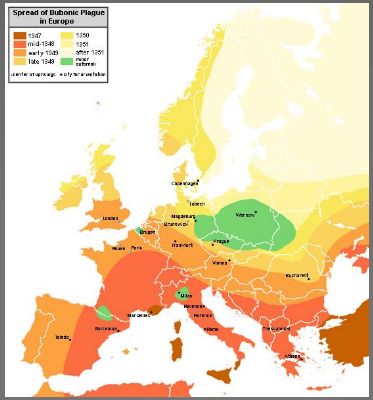
Notable Epidemic Outbreaks

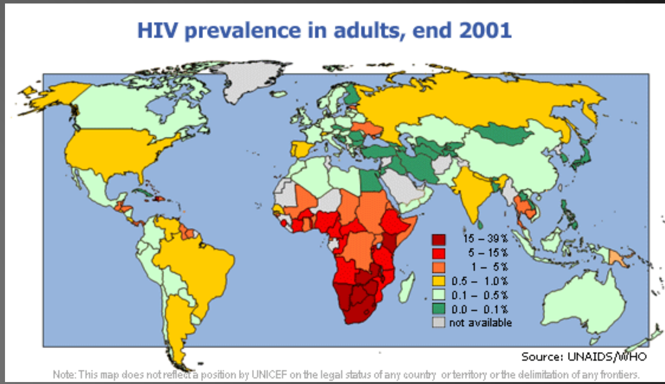
Epi + demos

upon people

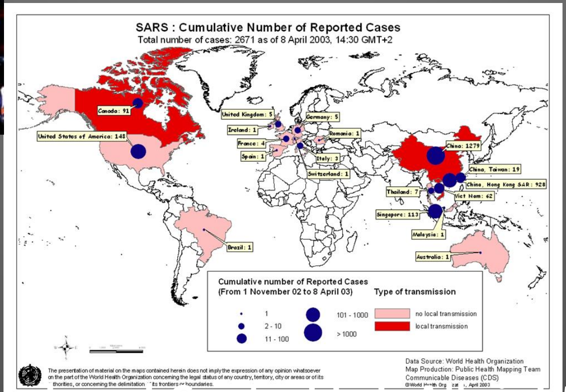
The Great Plague



HIV



SARS



H1N1 flu

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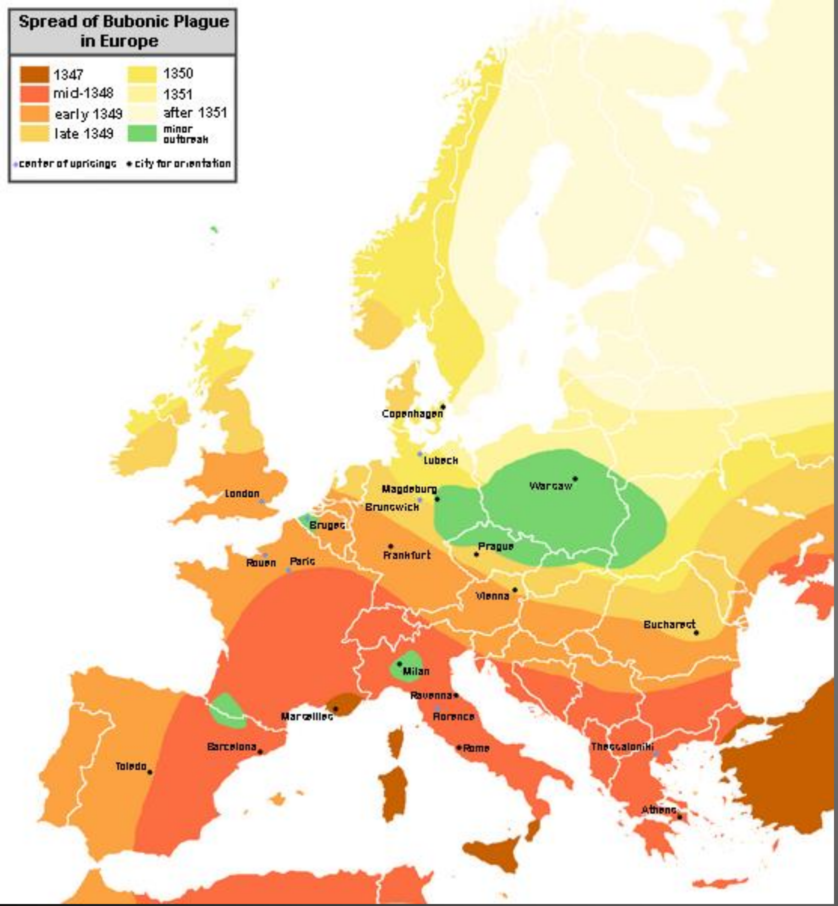
14th Century – The Great Plague



- **鼠疫大瘟疫：**墙上写个大大的“P”
- **六年时间：**1347至1353年
- **蔓延欧洲：**从意大利到西欧，而后北欧、波罗的海地区再到俄罗斯
- **夺走欧洲1/3人命：**2500万！
- **佛罗伦萨成为死城：**80%的人死去!

http://en.wikipedia.org/wiki/Black_Death
http://de.wikipedia.org/wiki/Schwarzer_Tod

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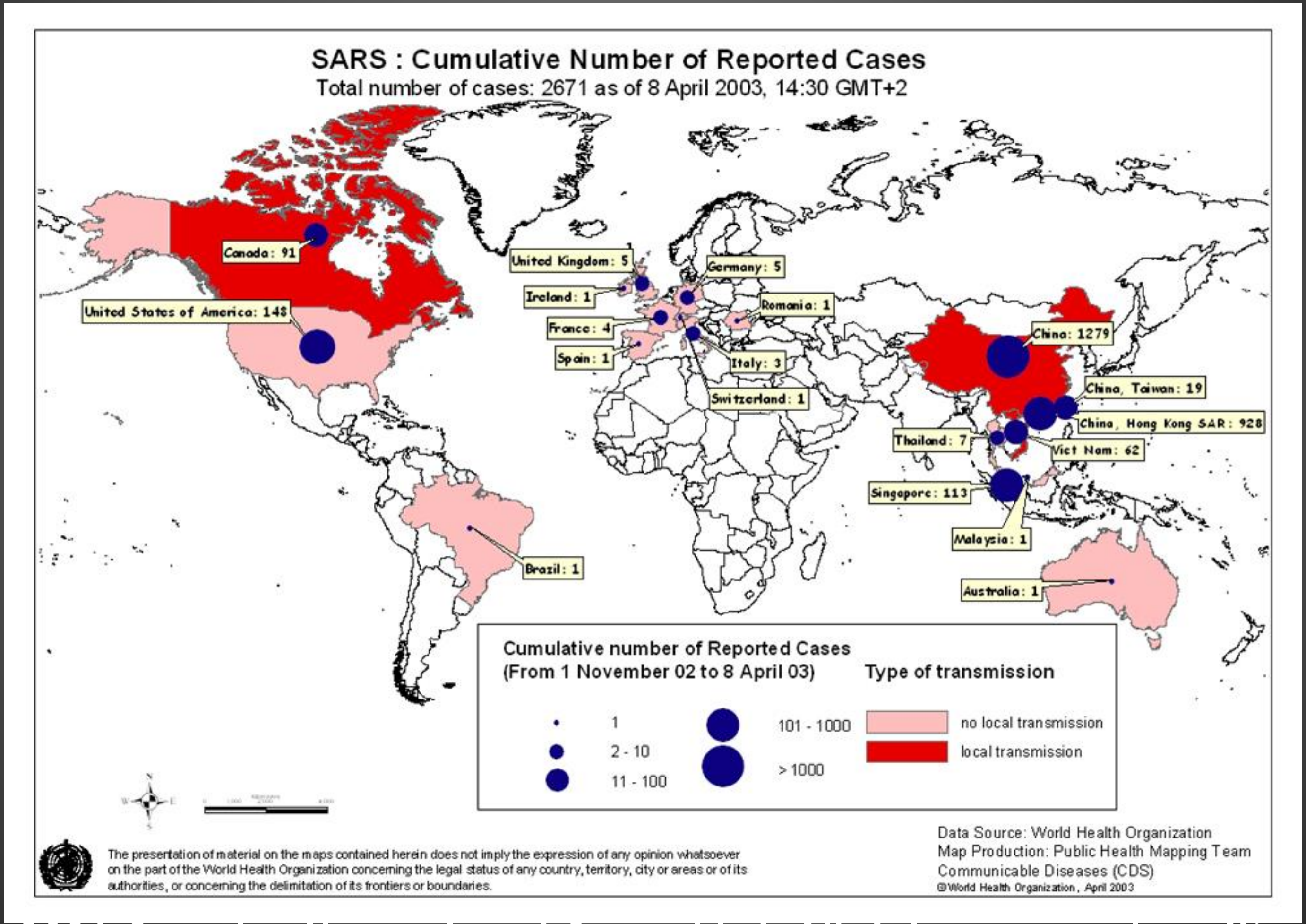
21th Century – SARS



Source: World Health Organization

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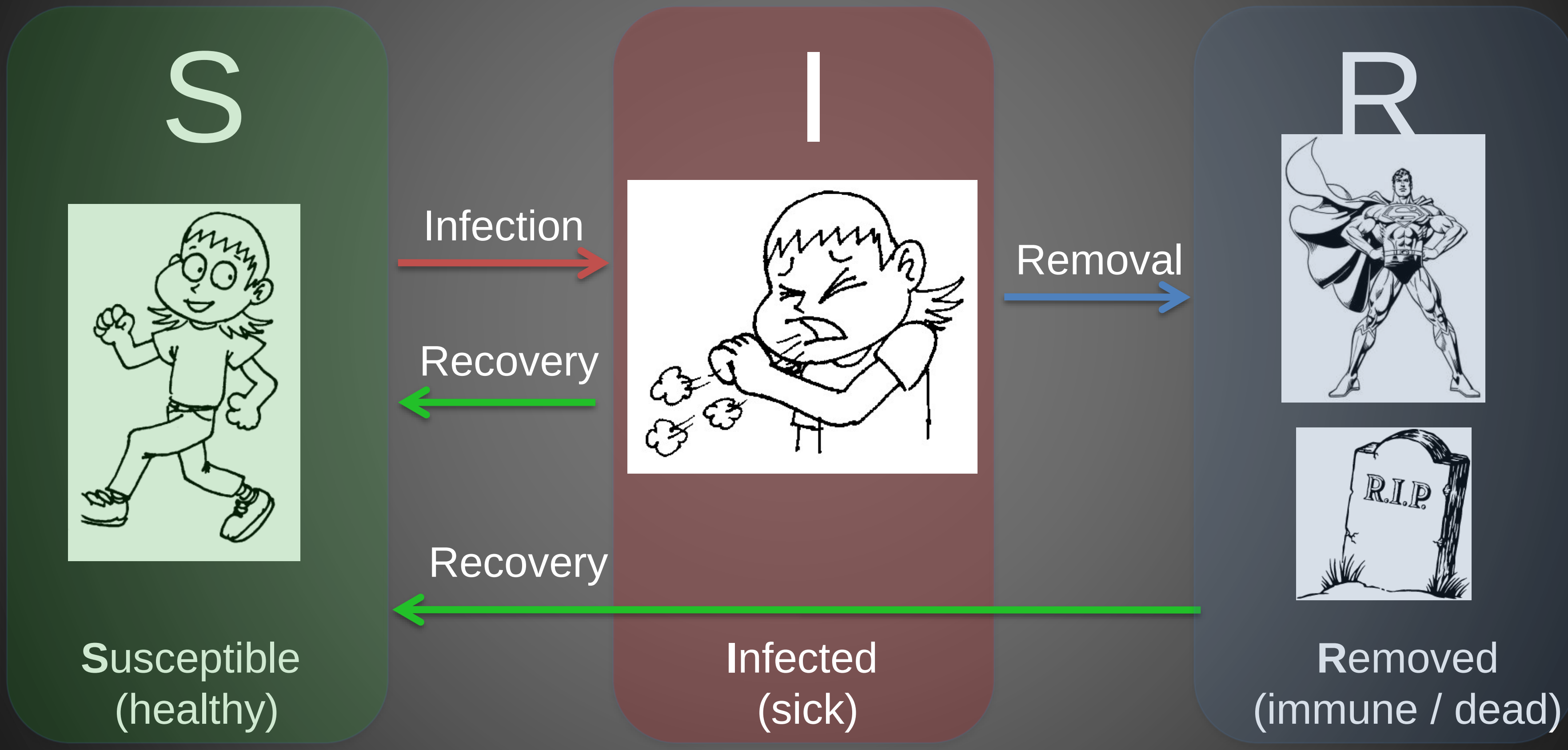
Classical Epidemic Models – Basic States



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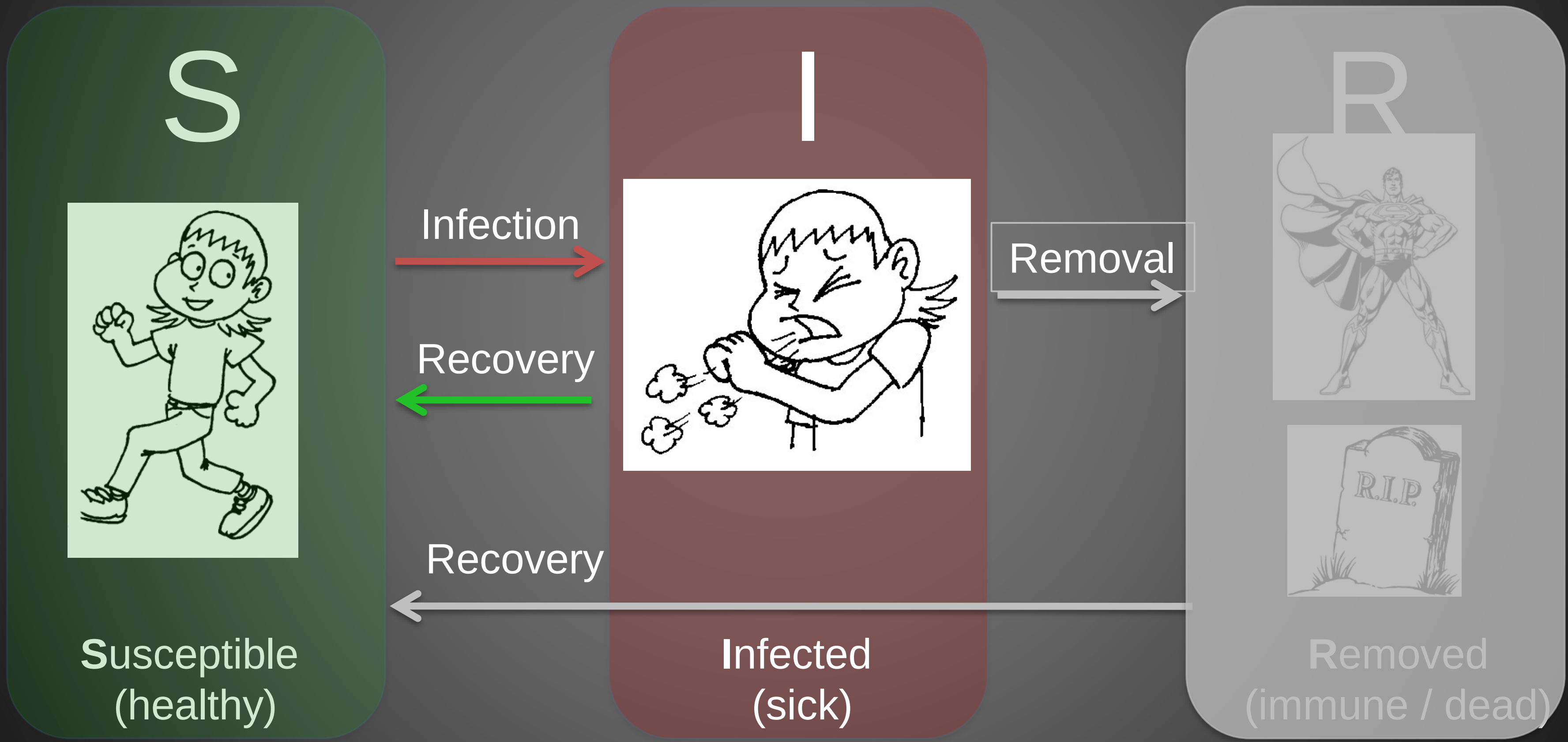
SIS Model



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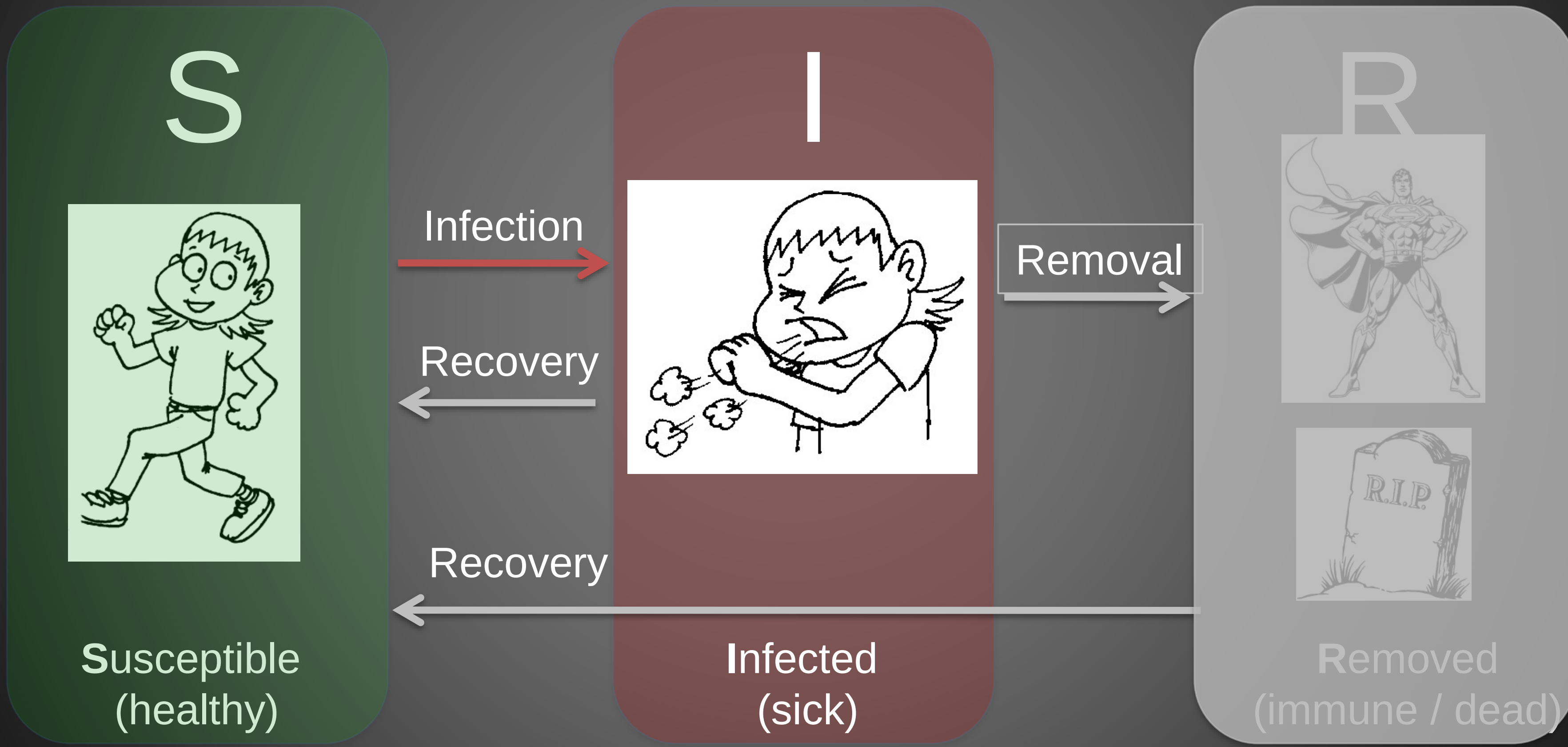
Simplest Model – SI



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Homogeneous Mixing Assumption

- 一个个体与任一其它个体接触的机会均等
- 一个易染个体与一个感染个体接触后被传染的概率为 β

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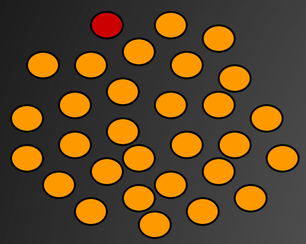
Simplest Model – SI



$$\frac{dI}{dt} = \beta \frac{I}{N} S \xrightarrow{s = S/N, \quad i = I/N} \frac{di}{dt} = \beta s i = \beta i(1-i)$$

$$i(t) = \frac{i_0 \exp(\beta t)}{1 - i_0 + i_0 \exp(\beta t)}$$

Logistic equation:

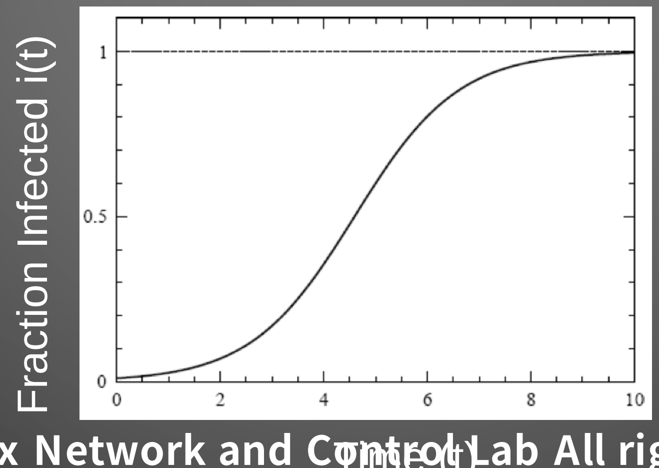
a basic model of population growth

If *i(t)* is small,

$$\frac{di}{dt} \approx \beta i$$

$$i \approx i_0 \exp(\beta t)$$

exponential outbreak



As i(t)→ 1.

$$\frac{di}{dt} \to 0$$

saturation

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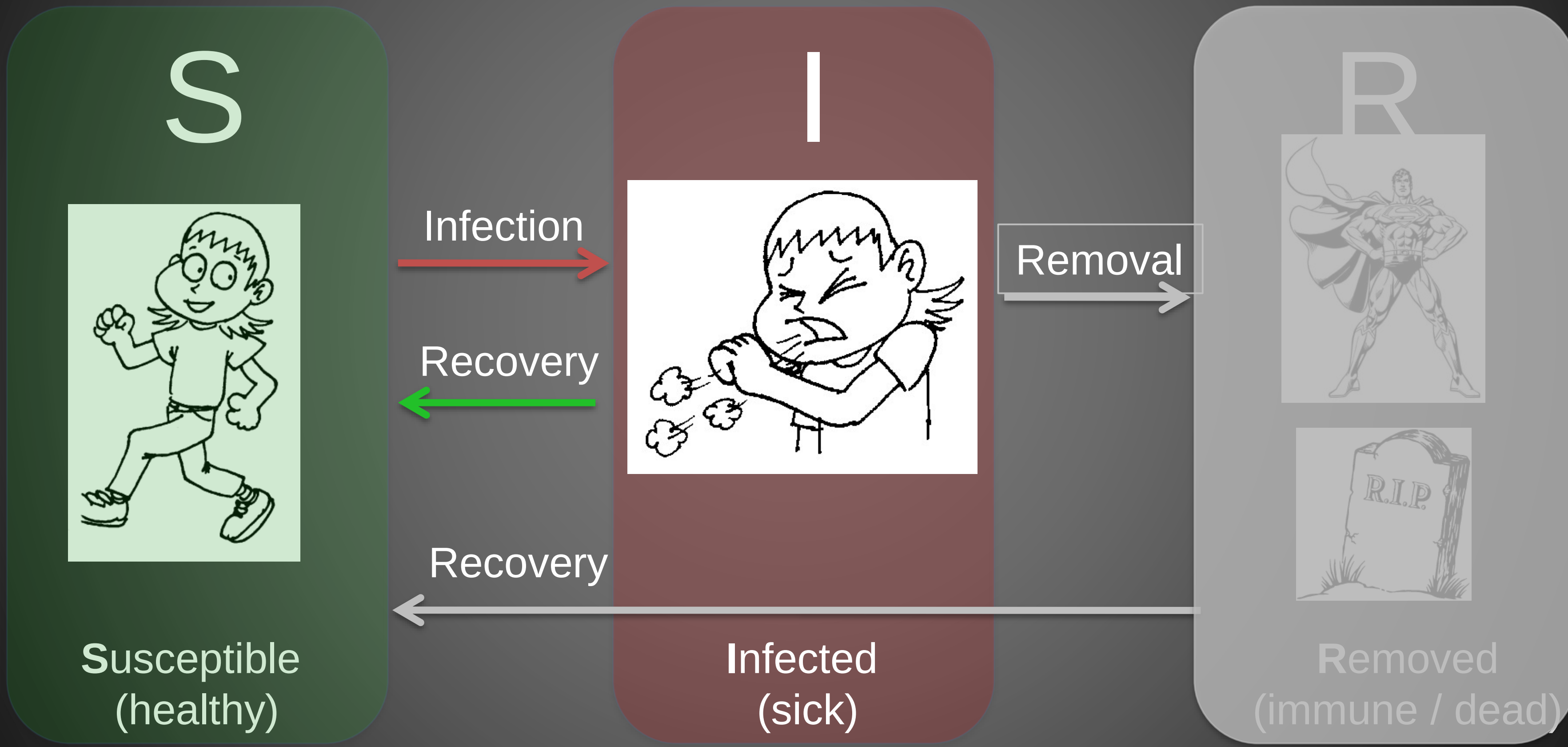
SIS Model


S
I
R
Infection
Recovery
Removal
Recovery
R.I.P
Susceptible
(healthy)
Infected
(sick)
Removed
(immune / dead)

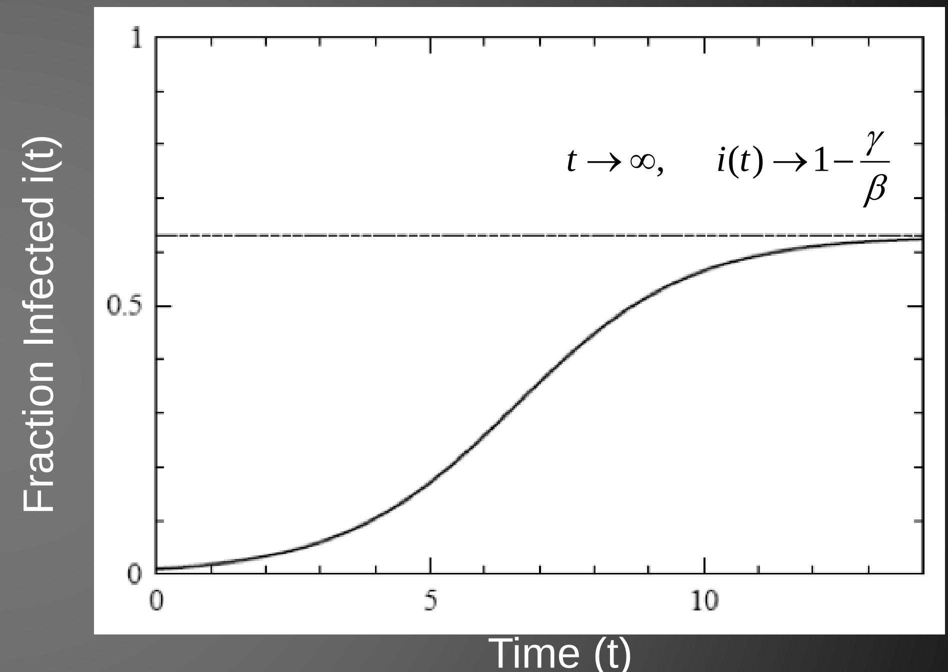
SIS Model

$$\frac{di}{dt} = \underbrace{\beta i}_{I}\underbrace{(1-i)}_{S} - \underbrace{\gamma i}_{I \to S} = i(\beta - \gamma - \beta i)$$

$$\frac{di}{i} + \frac{di}{1-\gamma/\beta - i} = (\beta - \gamma)dt$$

$$\ln(i) - \ln(1-\gamma/\beta - i) = (\beta-\gamma)t + c$$

$$i(t) = \frac{i_0(\beta-\gamma)e^{(\beta-\gamma)t}}{\beta - \gamma + \beta i_0 e^{(\beta-\gamma)t}}$$



If $\beta = \gamma, \quad i \to 0$

"Epidemic threshold"

SIS model: fraction infected individuals saturates below 1.

SIS Model:
Epidemic Threshold and Basic Reproductive Number

$$\frac{di}{dt} = \underbrace{\beta i}_{\mathrm{I}}\underbrace{(1-i)}_{\mathrm{S}} - \underbrace{\mu i}_{\mathrm{I}\to\mathrm{S}}$$

$$i(t) = \frac{i_0(\beta-\gamma)e^{(\beta-\gamma)t}}{\beta-\gamma+\beta i_0 e^{(\beta-\gamma)t}}$$

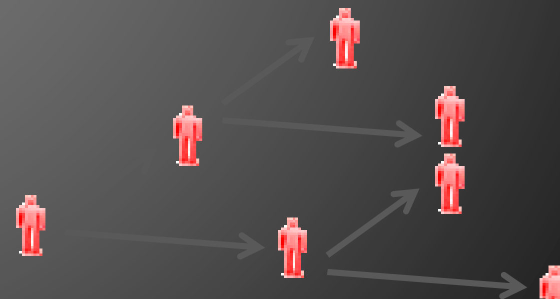
$\lambda \equiv \frac{\beta}{\gamma}$ **Basic reproductive number**

On average, how many infected individuals will be infected by one infected individual?

$\lambda = 1$

Epidemic threshold

$\lambda > 1$: Outbreak, $\lambda < 1$: Die out



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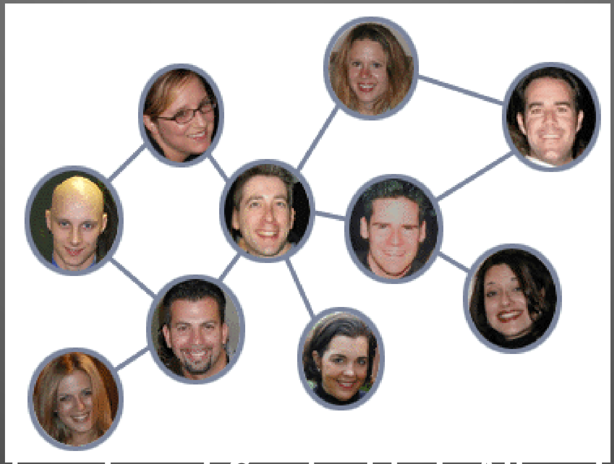
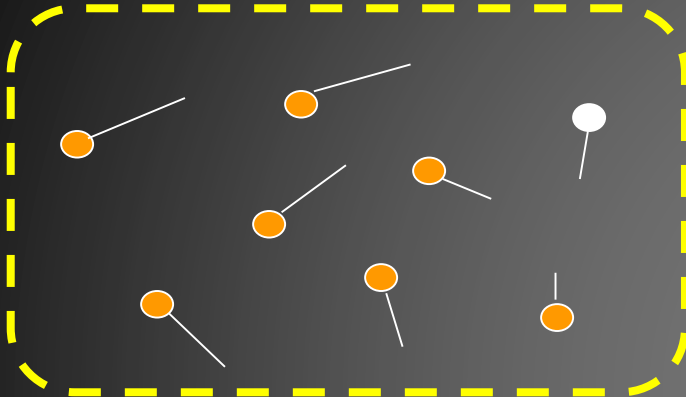
Epidemics on Networks

- **Homogenous mixing assumption** means that each individual can infect *any* other individual.
- In reality, epidemics spread along links in a network
- We need to explicitly account for the role of the network in the epidemic process.

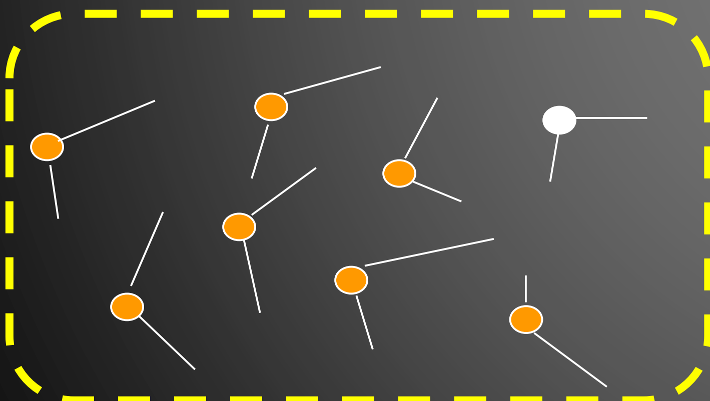


SIS model on a network: Degree based representation

Split nodes by their degrees



Class of nodes with degree *k=1*



Class of nodes with degree k=2

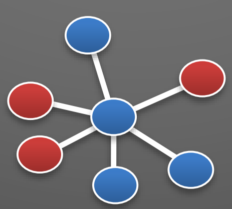
Fraction of infected nodes with degree *k*

$$\rho_k(t) \triangleq i_k(t) = \frac{I_k(t)}{N_k}$$

Fraction of infected nodes $i(t) = \sum_k P(k)\rho_k(t)$

SIS model $\frac{d\rho_k(t)}{dt} = -\rho_k(t) + \lambda k(1-\rho_k(t))\Theta(\rho_k(t))$

$\Theta_k(t)$: Density of infected neighbors of nodes with degree *k*

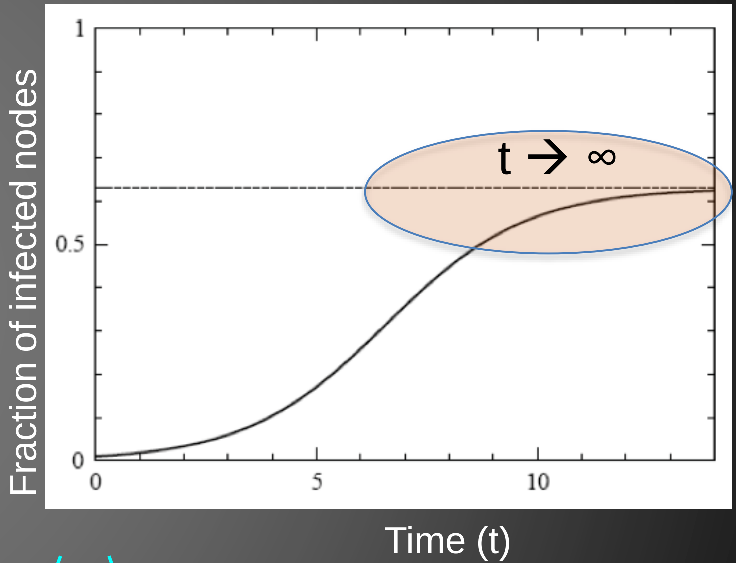


I am susceptible with *k* neighbors, and $\Theta_k(t)$ of my neighbors are infected.

SIS Model – Stationary state

$$\frac{d\rho_k(t)}{dt} = -\rho_k(t) + \lambda k(1-\rho_k(t))\Theta(\rho_k(t))$$

Stationary state condition: the number of new infections equals to the number of individuals who are cured



$$\rho_k = \frac{k\lambda\Theta_k}{1+k\lambda\Theta_k}$$

For an uncorrelated net. $P(s\,|\,k) = sP(s)/\langle k\rangle$

$$\Theta = \sum_s P(s\,|\,k)\rho_s = \frac{1}{\langle k\rangle}\sum_s sP(s)\rho_s$$

SIS Model – Stationary state

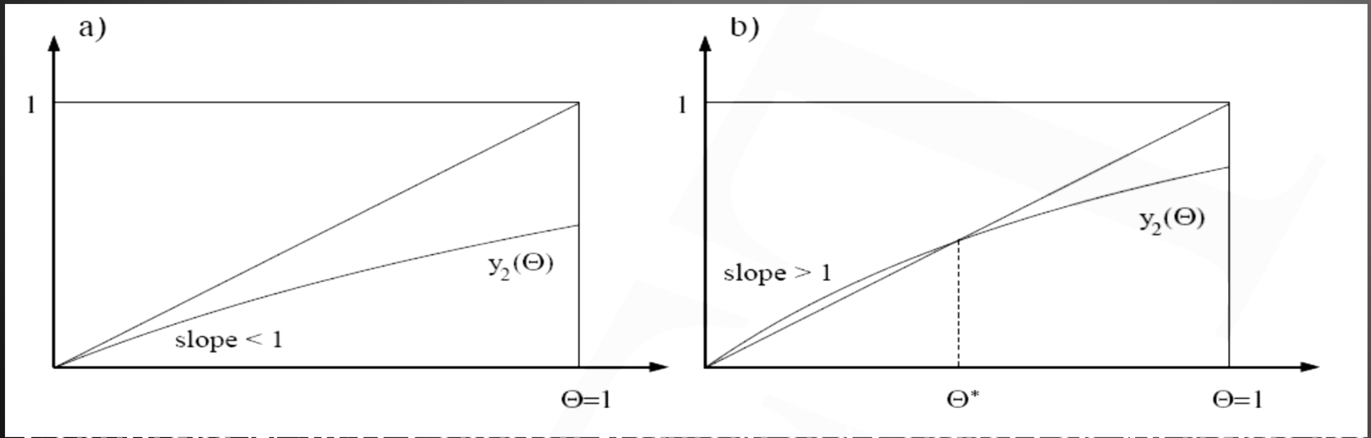
$$\Theta = \frac{1}{\langle k \rangle} \sum_s sP(s) \frac{\lambda s \Theta}{1+\lambda s \Theta}$$

$$\frac{d}{d\Theta}\left(\frac{1}{\langle k \rangle} \sum_s sP(s) \frac{\lambda s \Theta}{1+\lambda s \Theta} \right)\Bigg|_{\Theta=0} = \lambda \frac{\langle k^2 \rangle}{\langle k \rangle} \geq 1$$

$$= \frac{1}{\langle k \rangle} \sum_k kP(k) \frac{\lambda k(1+\lambda k \Theta) - \lambda k \Theta \lambda k}{(1+\lambda k \Theta)^2}\Bigg|_{\Theta=0} = \frac{1}{\langle k \rangle} \sum_k k^2 P(k) \lambda$$

$$\lambda_c = \frac{\langle k \rangle}{\langle k^2 \rangle}$$

$$\text{If } \langle k^2 \rangle \to \infty, \text{ then } \lambda_c \to 0$$



Pastor-Satorras & Vespignani, PRL 86, 3200(2001)

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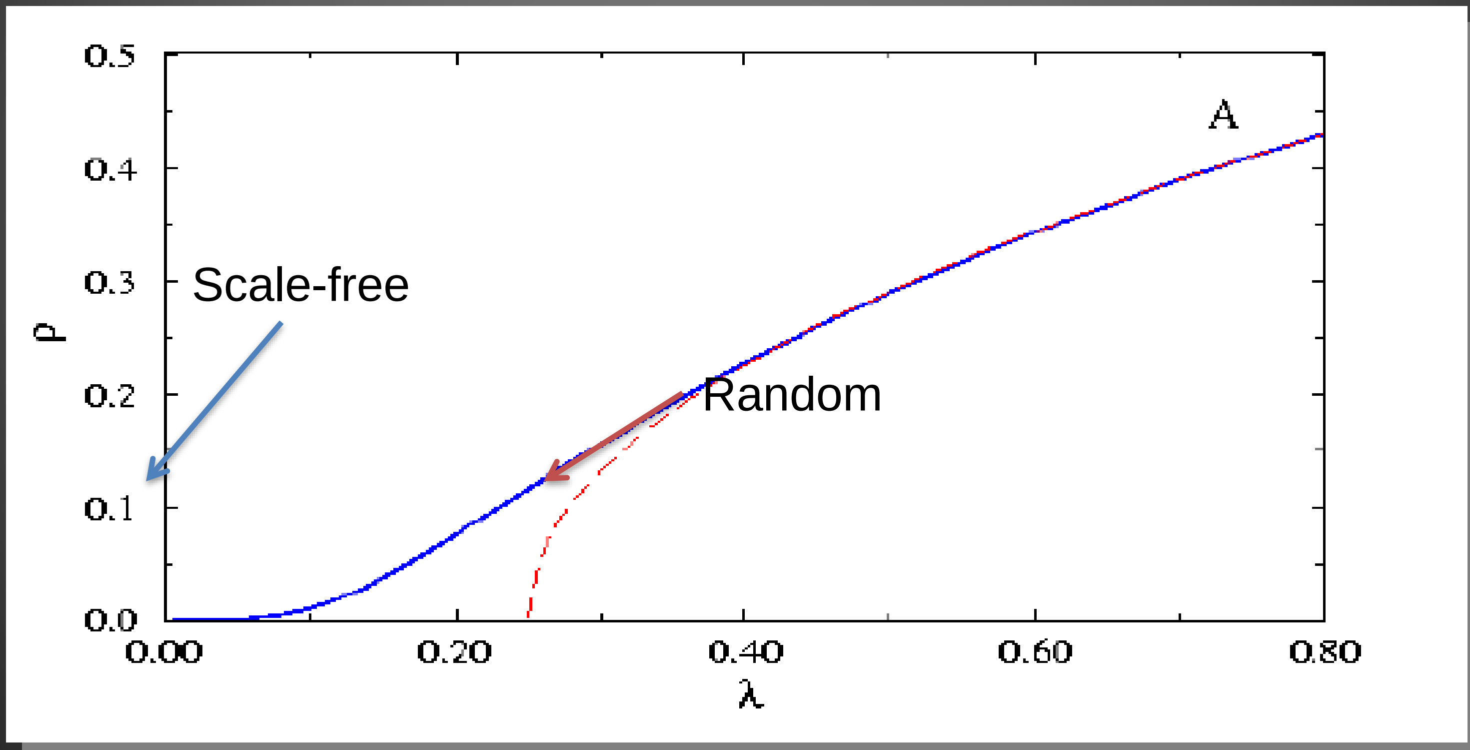
SIS Model

Vanishing Epidemic Threshold for Scale-Free Networks

$$\lambda_c = \frac{\langle k \rangle}{\langle k^2 \rangle} \qquad \text{If } \langle k^2 \rangle \to \infty, \text{ then } \lambda_c \to 0$$



Pastor-Satorras & Vespignani, PRL 86, 3200 (2001)

Immunization Strategies---
How to control the epidemic?

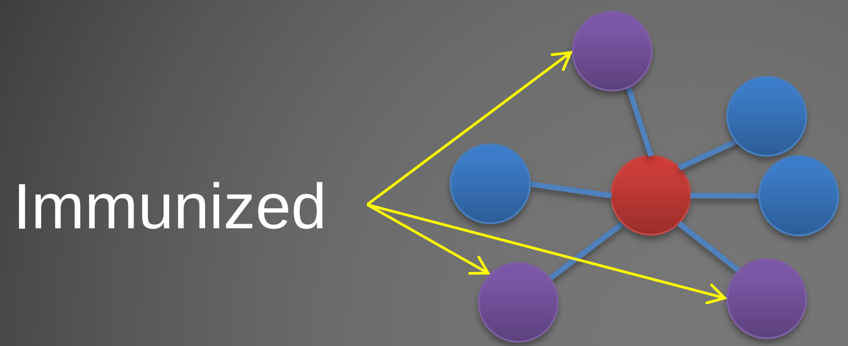
- **Transmission-reducing interventions:** face masks, gloves, washing hands – may reduce the transmission rate below the epidemic-causing critical rate
- **Contact-reducing interventions:** quarantining a patient, closing schools – make the network sparser, may increase the critical transmission rate
- **Vaccinations:** remove nodes from the network

Immunization Strategies---
How to control the epidemic?

- ***Who should be vaccinated for most effective control?***
- ***If it is too expensive to vaccinate everybody, then who should be vaccinated??***

Random Immunization

A density g individuals are randomly chosen to be immunized.



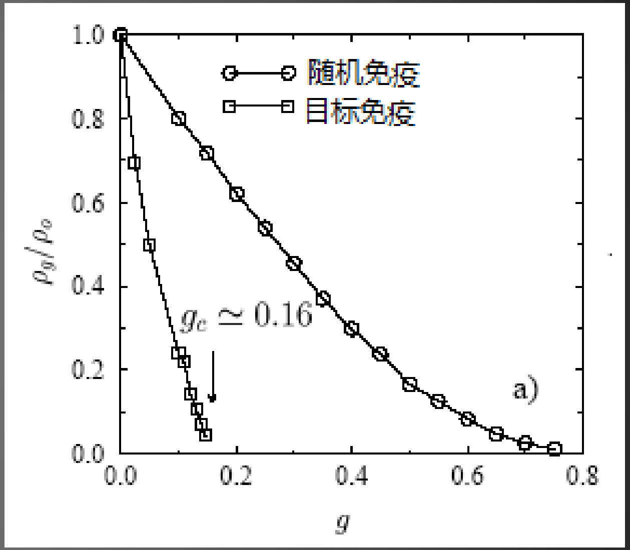
$$\lambda \to \lambda(1-g) \qquad \lambda(1-g_c) = \frac{\langle k \rangle}{\langle k^2 \rangle} \qquad g_c = 1 - \frac{1}{\lambda}\frac{\langle k \rangle}{\langle k^2 \rangle}$$

If $\langle k^2 \rangle \to \infty$, random immunization cannot prevent the outbreak

Targeted immunization

immunize all nodes with degree $k>k_0$

$$\lambda_c = \frac{\langle k \rangle}{\langle k^2 \rangle} = \frac{k_0 - m}{k_0 m}\left(\ln\frac{k_0}{m}\right)^{-1}.$$



As the hubs are removed, the $<k^2>$ term decreases, hence the epidemic threshold will go to higher values

In many cases, you cannot figure out who are the hubs.

Z. Dezso and A-L. Barabasi, Phys. Rev. E 65, 055103 (2002);
R. Pastor-Satorras and A. Vespignani, Phys. Rev. E 65, 036104 (2002)

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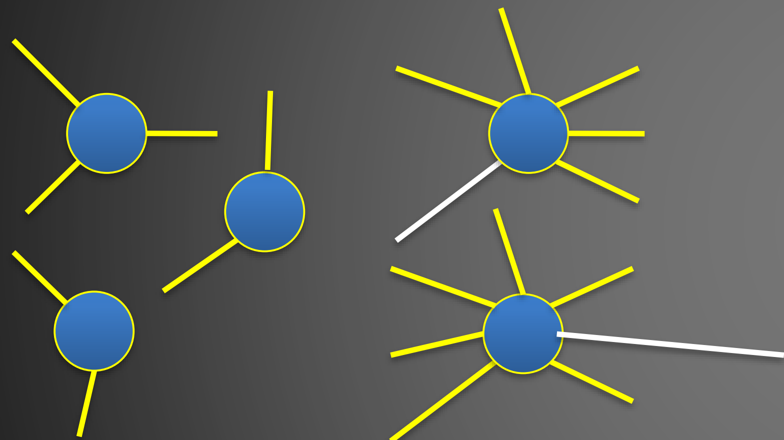
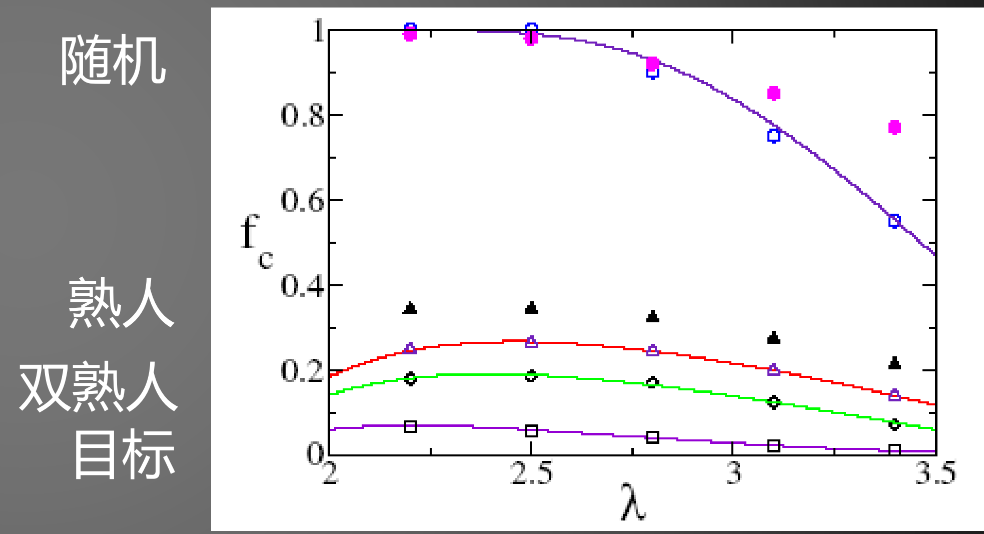
Acquaintance immunization

Select a random individual, then immunize one of its RANDOMLY CHOSEN FRIENDS.



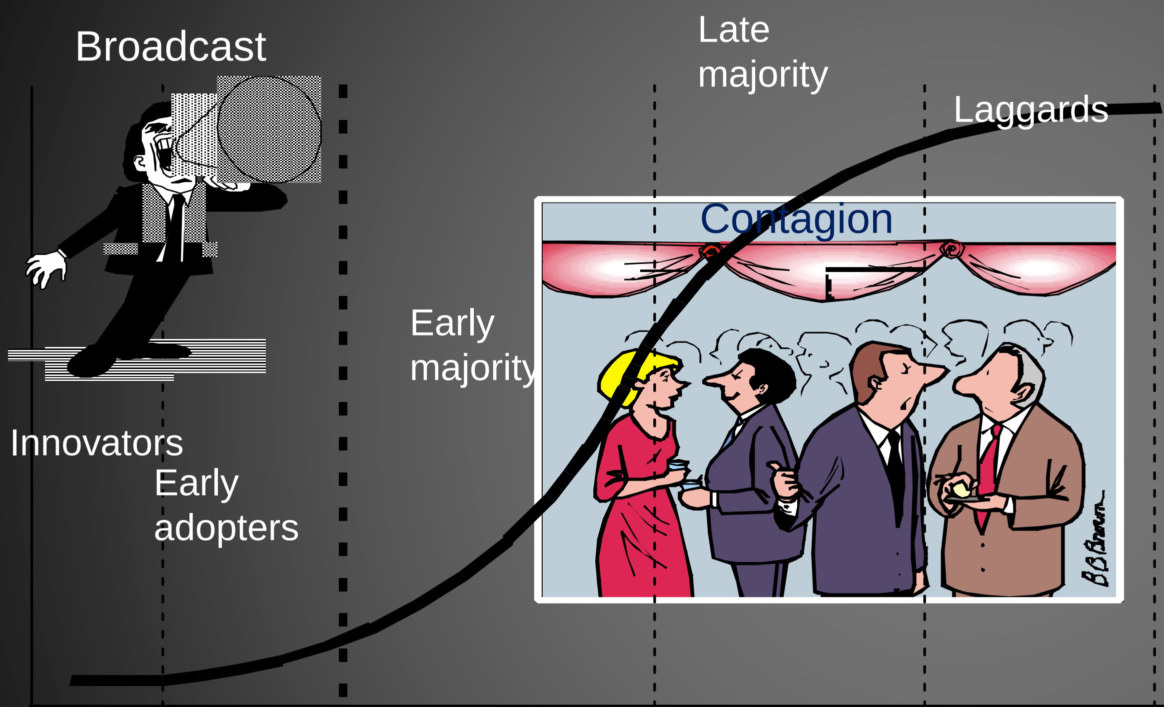
$\sim P(k)$ $\sim kP(k)$



If you follow an edge, you are likely to meet high-degree nodes!

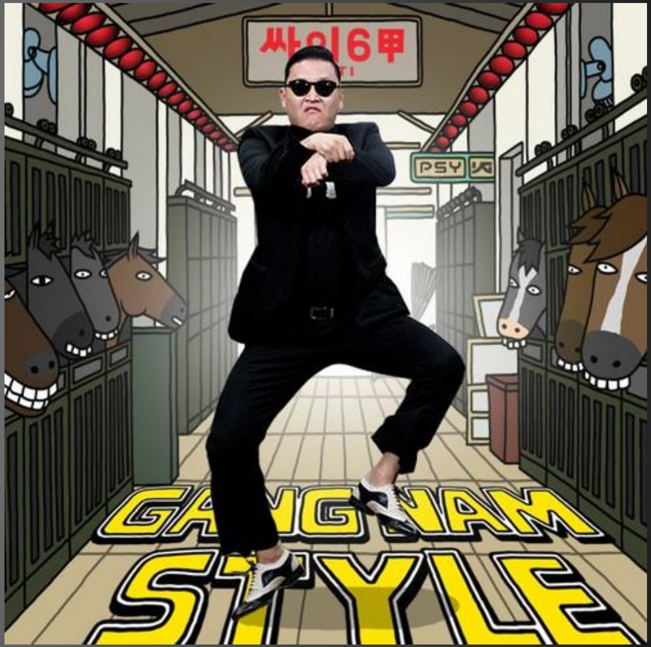
R. Cohen et al., PRL 91, 247901 (2003)

Diffusion of Innovation – The Adoption Curve



社会网络上的传播：热点能热多久？

- 太多热点导致绝大部分的生命力都很短：一个很快被另一个取代！
- 一些经典的诗、歌、文章、视频等却会一再重复发布、不断获得新生。



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Who Influences Who?
A Randomized Experiment

- **1.3 million Facebook users**
- **younger users are more susceptible to influence than older users**
- **men are more influential than women**
- **women influence men more than they influence other women**
- **married individuals are the least susceptible to influence in the decision to adopt the product offered.**

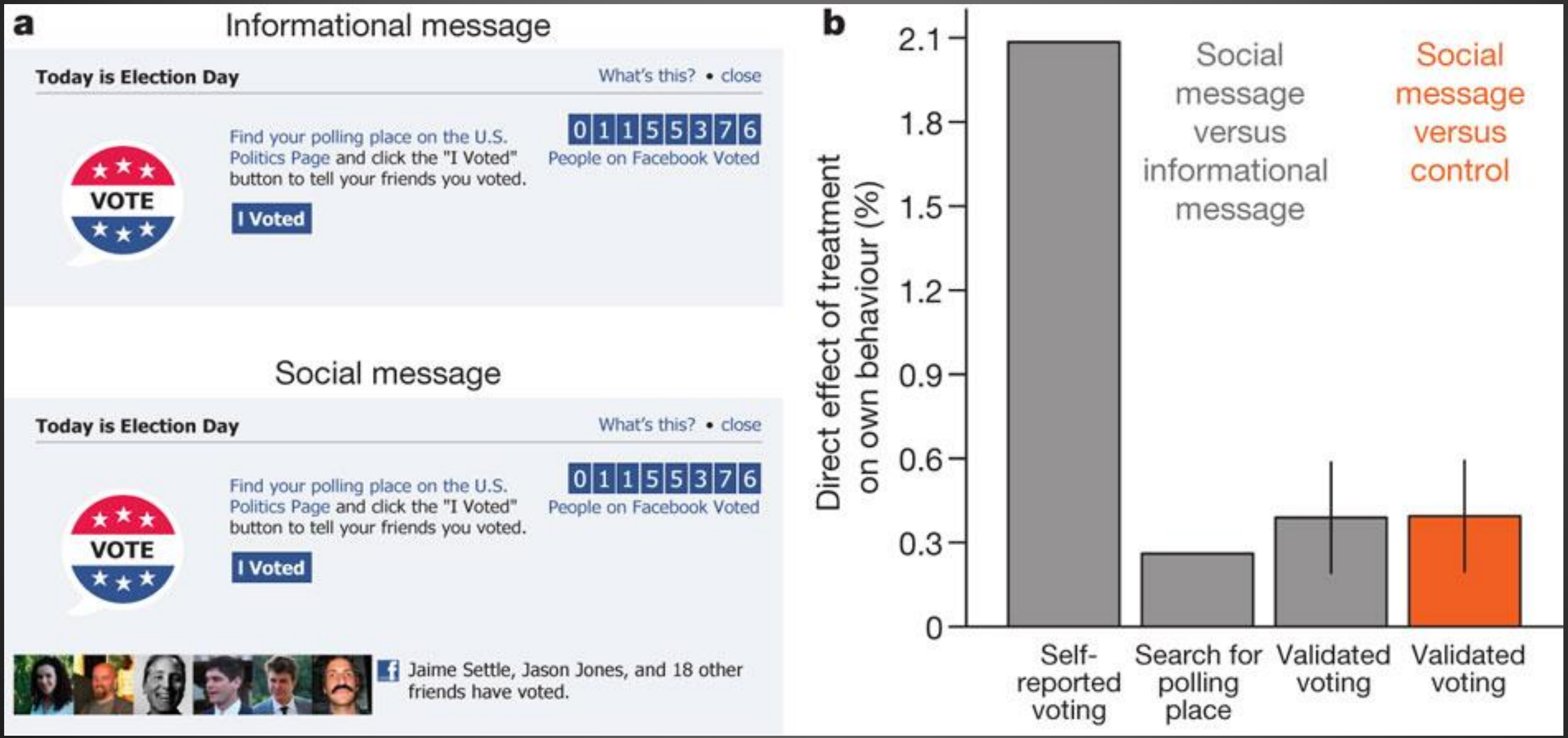
S. Aral & D. Walker. *Identifying Influential* and Susceptible *Members of Social Networks*: Science 20 Jul 2012; 337: 337-341

Who Influences Who?
A Randomized Experiment

- **influential individuals** are less susceptible to influence than **noninfluential individuals** and that they cluster in the network while susceptible individuals do not,
- which suggests that **influential people with influential friends** may be instrumental in the spread of this product in the network.

S. Aral & D. Walker. Identifying Influential and Susceptible Members of Social Networks, Science 20 Jul 2012; 337: 337-341

A 61-million-person experiment in social influence and political mobilization



- RM Bond et al. Nature **489**, 295-298 (2012), 13 Sept.

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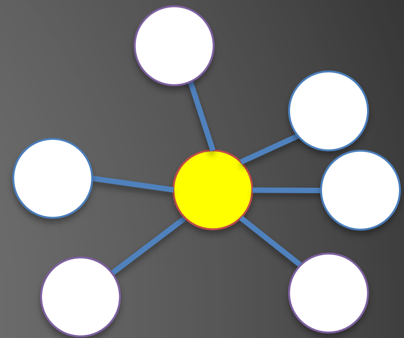
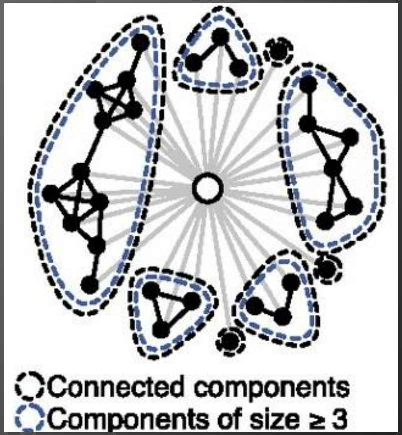
Structural diversity in social contagion

Ugander et al., PNAS 2012

- **Classical Assumption:** the probability that an individual is affected by the contagion grows monotonically with the size of his or her contact neighborhood
- **New Finding:** the probability of contagion is tightly controlled by the number of connected components in an individual's contact neighborhood





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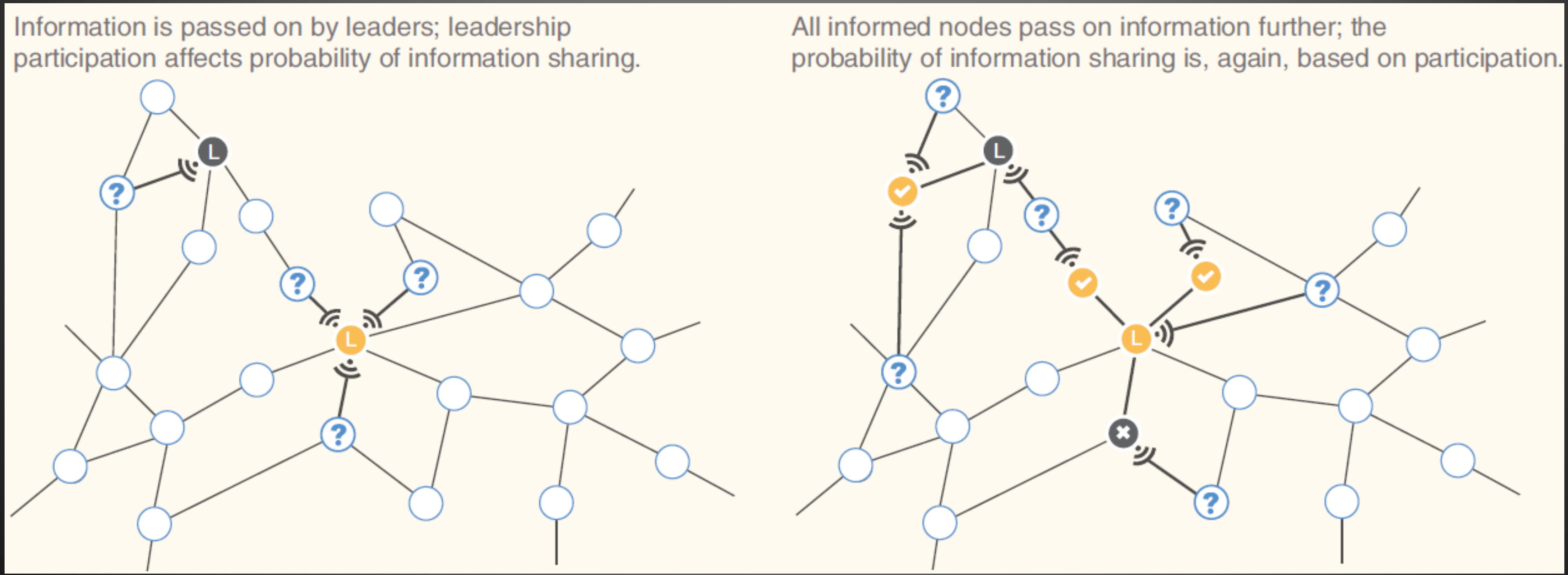
The Diffusion of Microfinance

Abhijit Banerjee et al. 341(6144): 363, 26 July, 2013

- How do the network positions of the first individuals to receive information about a new product affect its eventual diffusion?



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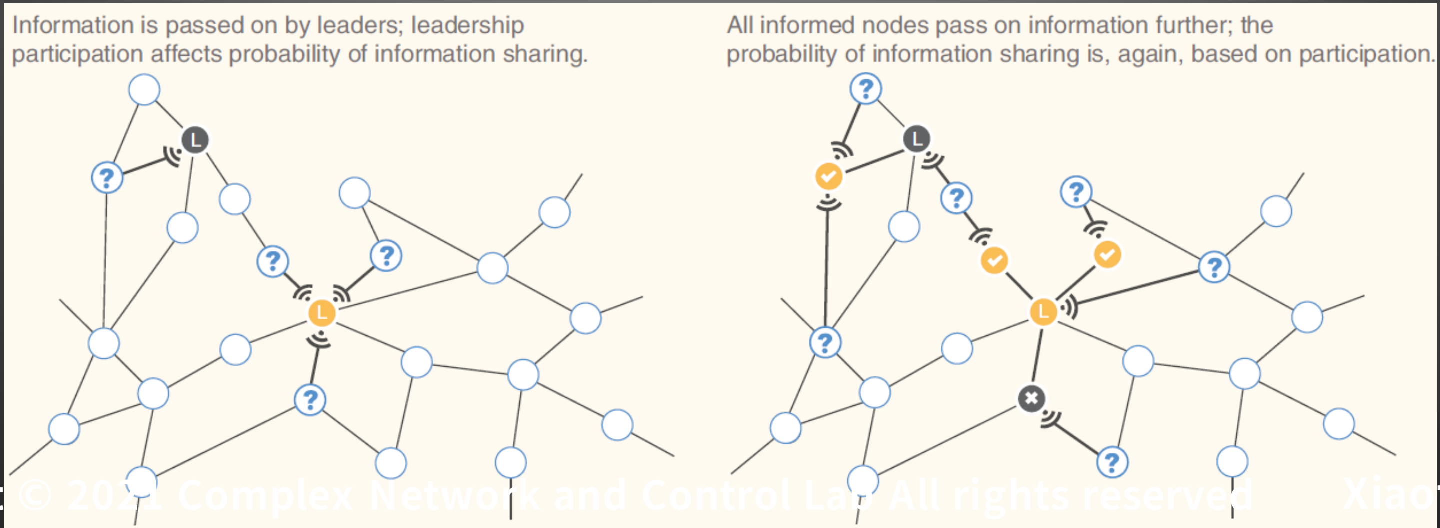
The Diffusion of Microfinance

- What are the factors that influence whether an individual chooses to adopt or purchase that product?

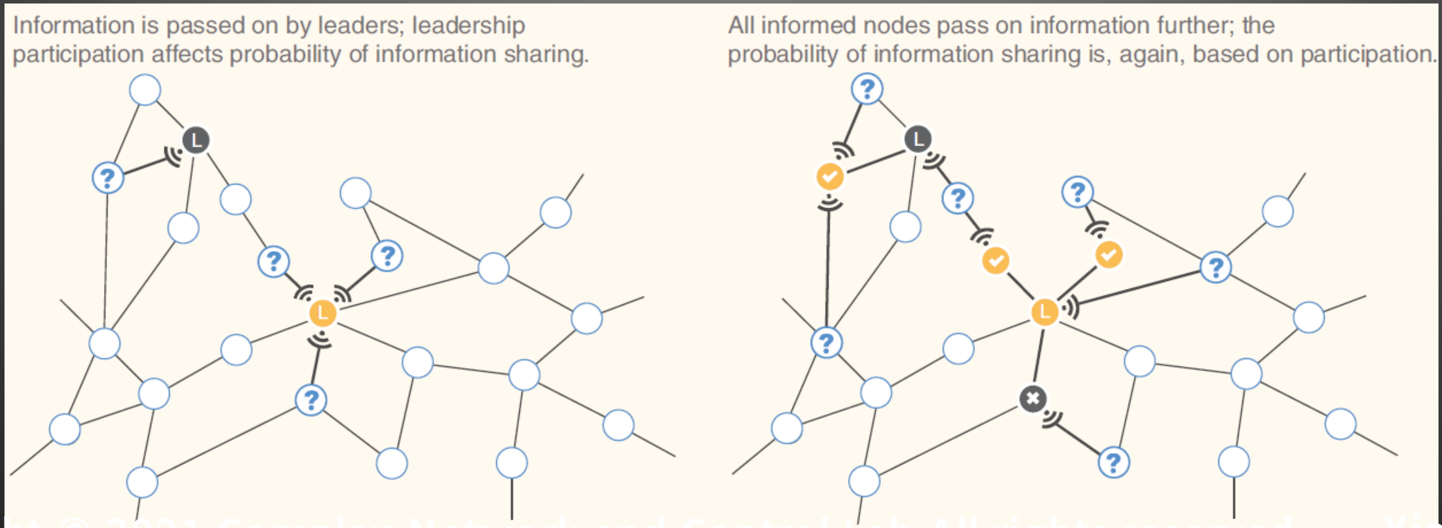
1. Individuals have to be aware of the product before they can adopt, which is more likely when more of their friends can tell them.
2. The adoption decisions of informed individuals might be influenced by the decisions of their friends.



The Diffusion of Microfinance

- To account for these factors, we developed a simple model of information diffusion that allows us to

1. distinguish information passing among neighbors from direct influence of neighbors' participation decisions
2. distinguish information passing by participants vs. nonparticipants.



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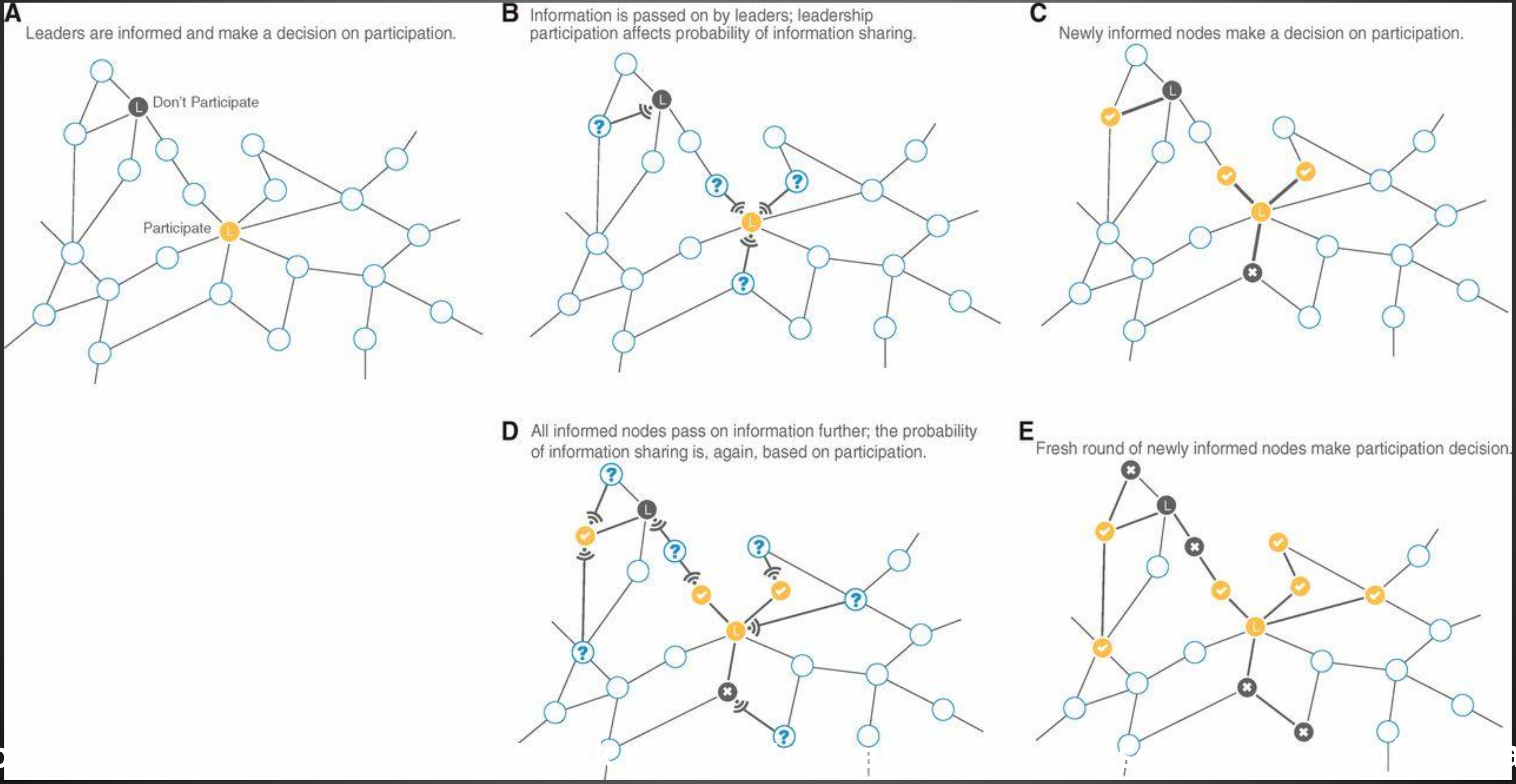
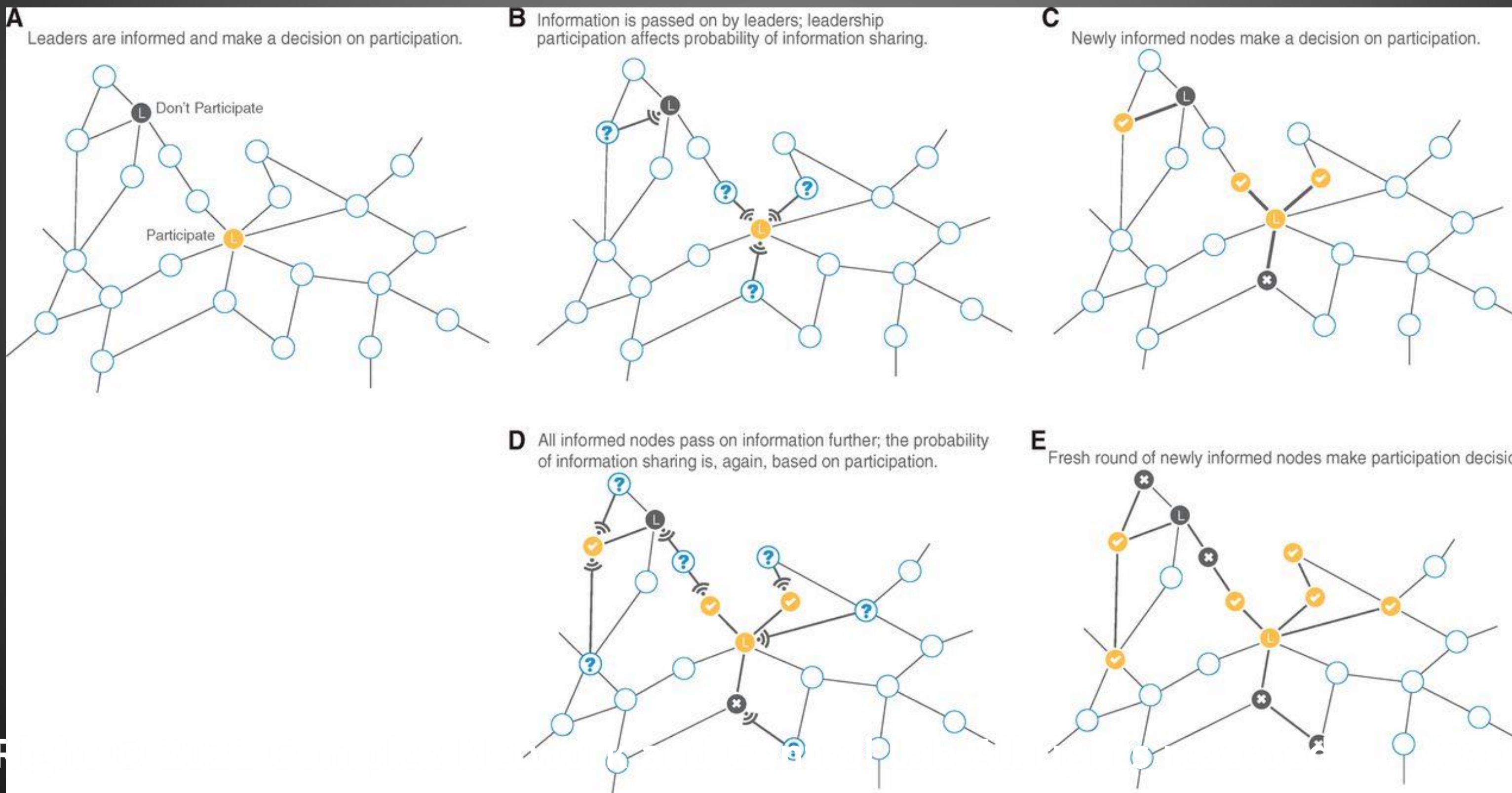
The Diffusion Model

1) An initial set of households is informed (injection points).
2) The initial households decide whether to participate.



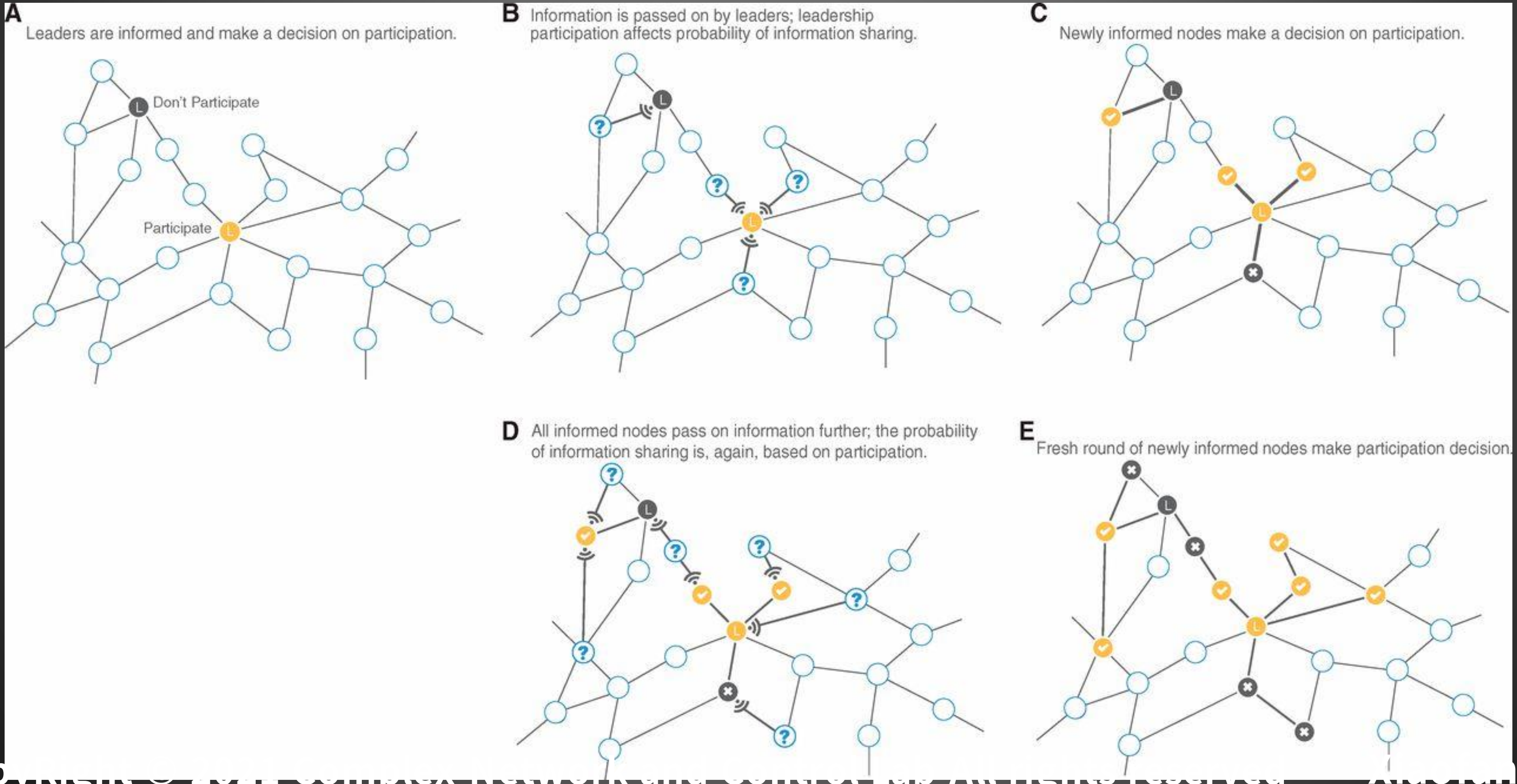
The Diffusion Model

3) In each subsequent period, households that have been informed in previous periods pass information to each of their neighbors, independently, with probability q^P if they are participants and with probability q^N if they are not.



The Diffusion Model

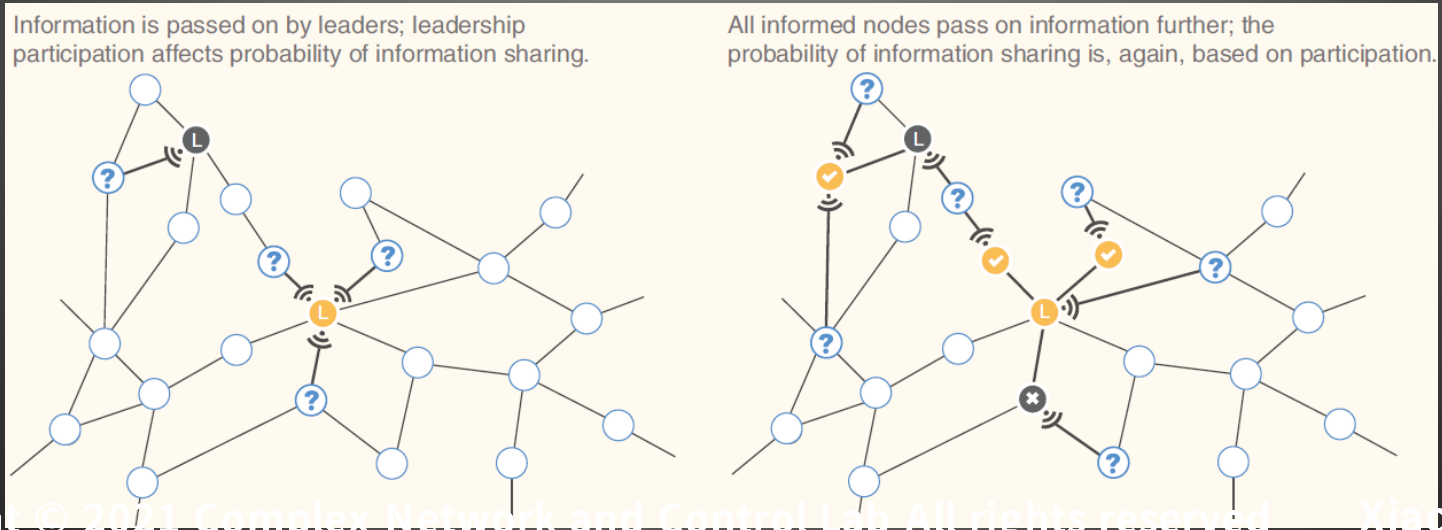
4) Newly informed households then decide whether to participate.
5) The process stops after *T* periods of information passing.



The Diffusion of Microfinance

Abhijit Banerjee et al. 341(6144): 363, 26 July, 2013

- A microfinance participant is seven times as likely to inform another household as a nonparticipant;
- Information transmitted by nonparticipants is important and accounts for about one-third of the eventual informedness and participation in the village because nonparticipants are much more numerous.



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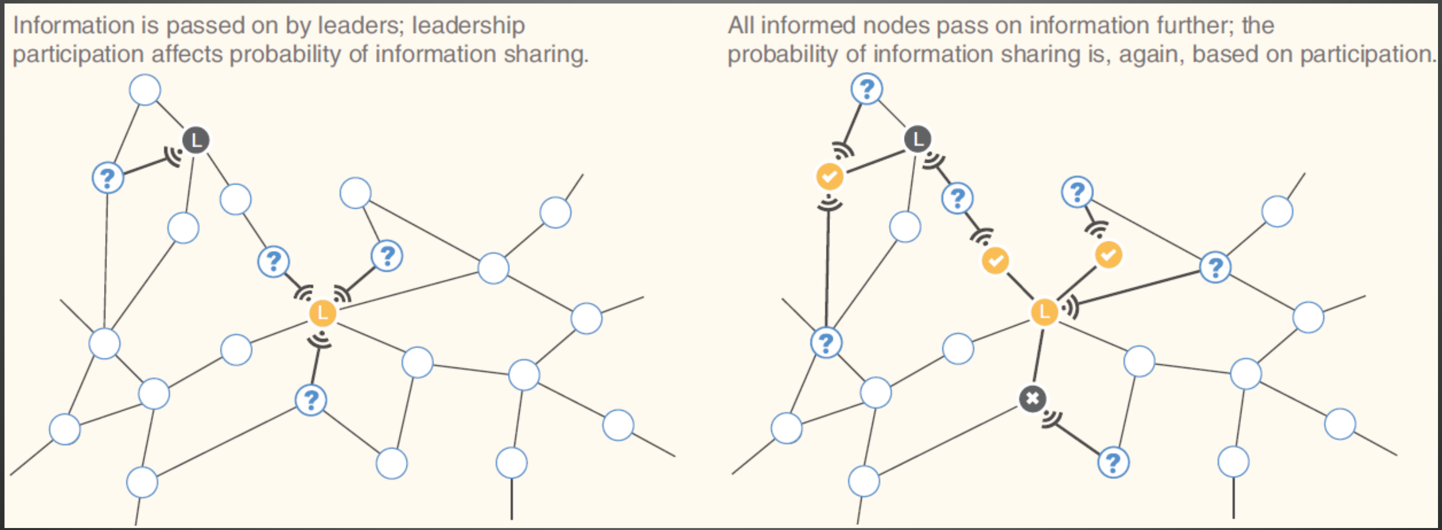
The Diffusion of Microfinance

Abhijit Banerjee et al. 341(6144): 363, 26 July, 2013

- How will the eventual participation in microfinance depend on whom the organization approach first?
- **Communication centrality** of a node: the fraction of nodes who would eventually participate if this node were the only one initially informed



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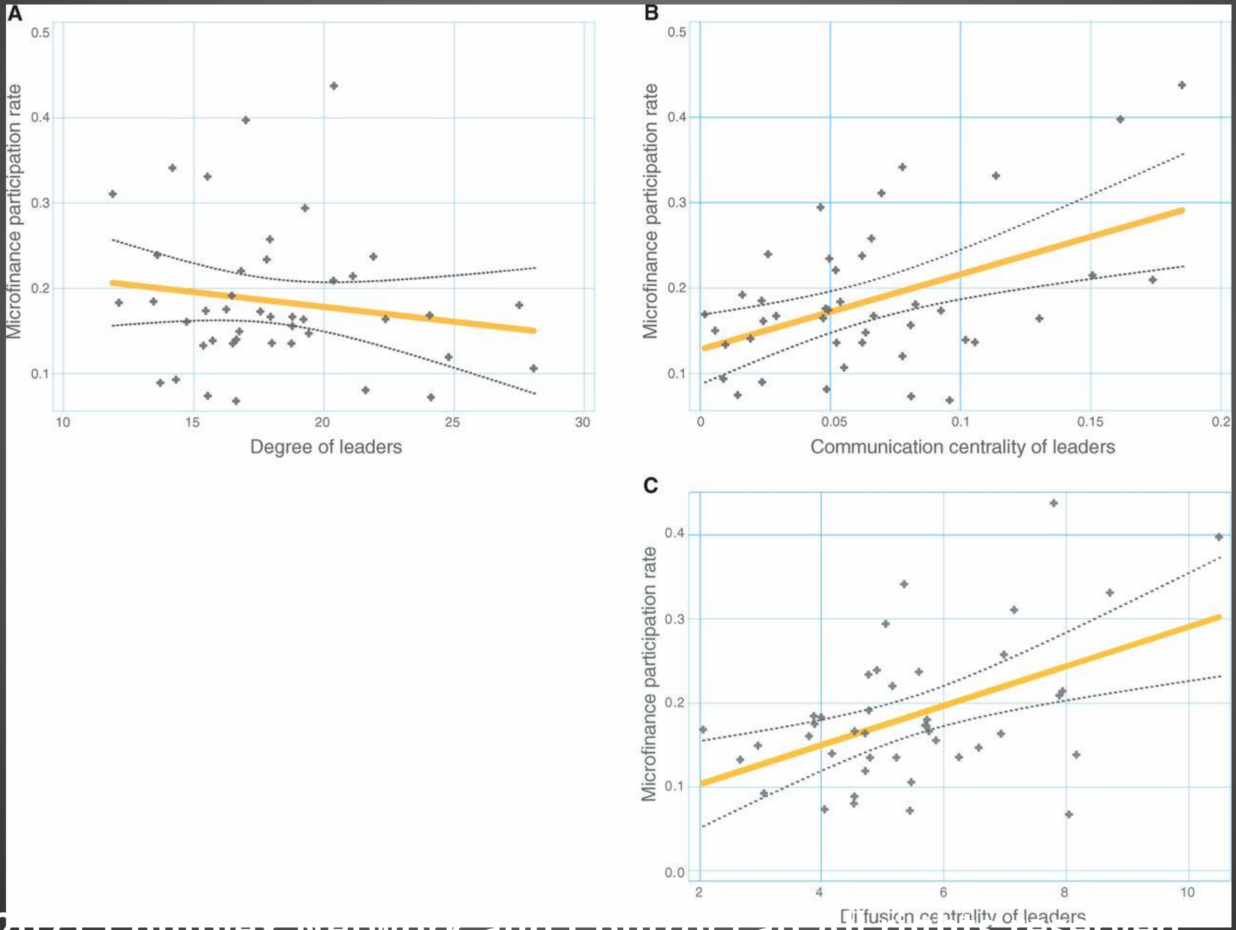
Diffusion centrality of node i as the *i*th entry of the vector

$$DC(A;q,T)=\left[\sum_{t=1}^{T}(qA)^t\right]\cdot 1$$

Assume adjacency matrix A, passing probability $q^N = q^P = q$, iterations T

- DC of node i corresponds to the expected total number of times that all nodes taken together hear about the opportunity.
- If $T = 1$, DC is proportional to degree centrality.
- As $T \to \infty$ it becomes proportional to either Katz-Bonacich centrality or eigenvector centrality, depending on whether q is smaller than the inverse of the first eigenvalue of A or exceeds it, respectively.
- In the intermediate region of T, DC differs from existing measures.

Participation village-by-village as a function of the average centrality of the leaders in the village



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互联网时代的羊群效应

大众点评的餐厅评价

豆瓣的电影评价

购物网站的商品评价

差评师的影响有多大

- 事先为101281篇网上文章随机给好评或差评，并与对照组相比
- **事先给好评有引导性：**读者给好评的可能性提高了32%
- **事先给差评没有引导性：**对文章最后的评分几乎没有影响

Lev Muchnik, Sinan Aral, and Sean J. Taylor, Social Influence Bias: A Randomized Experiment, Science 9 August 2013: 341 (6146), 647-651.

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Epidemiological modeling of online social network dynamics

SIR model

$$\dot{S} = -\frac{\beta IS}{N}$$

$$\dot{I} = \frac{\beta IS}{N} - \gamma I$$

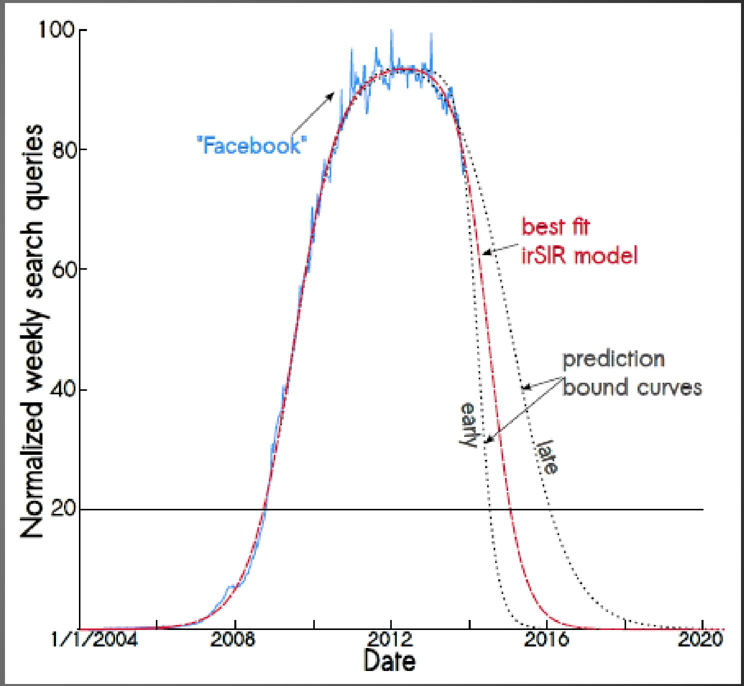
$$\dot{R} = \gamma I$$

irSIR model

$$\dot{S} = -\frac{\beta IS}{N}$$

$$\dot{I} = \frac{\beta IS}{N} - \frac{\upsilon IR}{N}$$

$$\dot{R} = \frac{\upsilon IR}{N}$$



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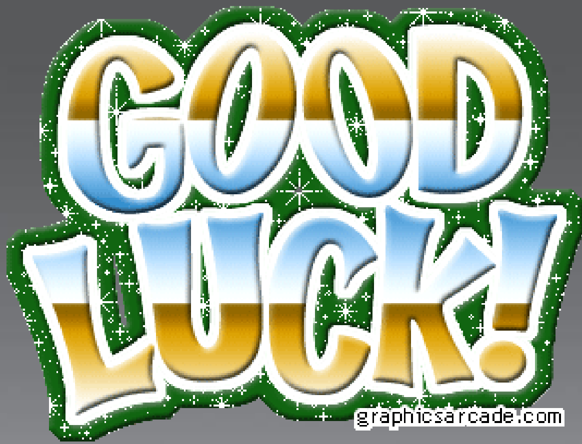

Thank You !

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