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Review

Pinning control of complex networked systems: A decade after and beyond ☆

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ABSTRACT

In practice, directly control every node in a dynamical networked system with a huge number of nodes might be impossible or unnecessary; therefore, pinning control is a desirable approach. This paper surveys advances in pinning control approaches to making a dynamical networked system have a desired behavior. For a network with fixed topology, we review the feasibility, stability and effectiveness of pinning control. We then focus on pinning-based consensus and flocking control of mobile multi-agent networked systems. One of the main challenges with consensus and flocking control is that the topology of the corresponding dynamical network is time-varying, which depends on the states of all the agents in the network. Looking forward to the next decade, we expect to have a much deeper understanding of the relationship between the effectiveness of pinning control and the structural properties of a complex network, which may result in better control of large scale networked systems.

Contents

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1. Introduction

The turn of the century was also a turning point in network science. The two ground-breaking papers Watts and Strogatz (1998) and Barabási and Albert (1999) published at the very end of the last century revealed the small-world and scale-free features of complex networks, which have stimulated an avalanche of research on complex network topology and modeling. From the control point of view, a nature step is to understand the influence of network structure on network behavior and consequently to find effective ways to improve network performance.

Synchronization is one of the ubiquitous dynamical phenomena in nature and there have been a lot of researches on synchronization in networks with regular topologies in 20th century Kuramoto (1984) and Wu and Chua (1995). Since 2002, there has been much interest in investigate the synchronizabilities of networks with complex topologies Wang and Chen (2002a, 2002b, 2003) and Barahona and Pecora (2002).

Control will be a necessary means for guiding or forcing the network to achieve desired synchronization if a given network is not synchronizable or if the synchronized state is not the desired state. In practice, directly control every node in a dynamical network with a huge number of nodes might be impossible or unnecessary. Therefore, pinning control strategy, that is, to achieve the goal of control by directly adding control inputs to a fraction of nodes selected from the network, is very important. Wang and Chen (2002c) firstly studied the control of a scale-free dynamical network to its equilibrium via pinning. Li, Wang, and Chen (2004) further specified the stable conditions and bridged the pinning control with virtual control propagation to explore the efficiency of selective pinning strategies. Since then, many researchers have contributed to the fruitful understandings in this topic, including complete synchronization, cluster synchronization, selective strategies, controllable regions, and control methodologies for different specific scenes Su and Wang (2013).

Subsequently, the idea of pinning control has also been applied to coordinated control of multi-agent systems. One of the main challenges here is that the topology of the corresponding dynamical network is time-varying which depends on the states of all the agents in the network. Furthermore, connectivity of the initial network cannot guarantee connectivity of the network all the time. One way to overcome this difficulty is to assume that there is a real or virtual leader and that every agent is an informed agent which has the information of the leader so that a navigational feedback term could be added to every agent. In this way, all agents could remain cohesive and asymptotically move with the same desired velocity no matter whether the initial network is connected. However, this assumption is in contrast with some nature examples and may be difficult to implement in engineering applications.

The recent research results in pinning control for synchronization of complex dynamical networks, and consensus and flocking of multi-agent systems via pinning mainly include the following topics Su and Wang (2013):

(i) Stability conditions for synchronization of complex dynamical networks via pinning. Various methods have been investigated towards finding sufficient conditions for global or local asymptotic stability. Pinning control was firstly investigated in Wang and Chen (2002c) for the local stabilization of a scale-free dynamical network. Li et al. (2004) further investigated both the global and local stabilization of complex dynamical networks via the pinning control strategy. Sufficient conditions were presented to guarantee synchronization by pinning only one node Chen, Liu, and Wu (2007). Pinning control based on the Lyapunov V-stability approach was investigated, which converts the network stability problem into the measurement of the negative definiteness of a matrix that characterizes the topology of the network Xiang and Chen (2007). A low-dimensional condition without the pinning controllers involved was proposed in Yu, Chen, and Lu (2009). By using the local pinning control algorithm, some sufficient conditions concerning the global stability of controlling a complex network with digraph topology to a homogeneous trajectory of the uncoupled system were derived in Lu, Li, and Rong (2010).

(ii) Selective strategies of pinning control. The efficiency to fulfill different specifications varies from the network considered and the pinning strategies chosen. Selective pinning strategies play an important role to achieve better efficiency. Wang and Chen (2002c) showed that pinning the high-degree nodes is more efficient than pinning randomly chosen ones in a scale-free network. Li et al. (2004) further proposed the concept of virtual control and showed why a high-degree pinning scheme is more effective than a random pinning one in a scale-free network. In order to enhance the synchronization performance, it was shown that the nodes adjacent to the highest degree node are good candidates to be pinned Turci and Macau (2011). Miao, Rong, Tang, and Fang (2008) showed that a max-degree pinning scheme is better than a random pinning one in a disassortative network, while the combination of the two schemes is more effective in an assortative network. Zou and Chen (2007) illustrated that pinning the small nodes is better than pinning the big nodes when the portion of pinned nodes is relatively large in an unweighted symmetrical scale-free network, but pinning the big nodes is, in fact, always better than pinning the small ones in normalized weighted scale-free networks. The investigation in Rong, Li, and Lu (2010) showed that pinning the nodes with high betweenness centralities is usually more effective than pinning the ones with high degrees. It was illustrated that for any network structure, the pinning control performance was maximized via uniform pinning of all the network nodes Porfiri and Fiorilli (2009). For a scale-free directed dynamical network, it was shown that pinning a node with the largest Control-Rank is much more effective than pinning a node with the largest out-degree Lu and Wang (2008). In Tang, Ng, and Jia (2010), the authors modified the traditional degree-based strategy, and used a decrease-and-conquer approach to assign the best control strengths to the pinned highest-degree nodes.

(iii) Controllable regions. By evaluating the pinning controllable regions of a given network, one could judge the effectiveness of a selective pinning strategy. Based on the eigenvalue analysis, Zhan, Gao, Wu, and Xiao (2007) obtained the controllable regions directly in the control parameter space for both the diagonal coupling and nondiagonal couplings. An extension of the master stability function approach was proposed for the controllability of networks under pinning control, in which the controllable regions were characterized by the coupling gain, the control gain, and the number of pinned nodes Sorrentino, Bernardo, Garofalo, and Chen (2007). Several manageable criteria were proposed, and the effects of the network topology, the location and number of pinned nodes, and the nodes intrinsic dynamics on the global pinning-controllability were analyzed Porfiri and Bernardo (2008). In Xiang, Chen, Liu, Chen, and Yuan (2009), the stable regions of the coupled network were clarified and the eigenvalue distribution of the asymmetric coupling and control matrices were specified for asymmetric networks.

(iv) Control methodologies. In order to adapt to different specifications, many methodologies in control theory have been introduced for pinning control, including adaptive control, intermittent control, impulsive control, stochastic control, finite-time control and time-delay control. By introducing local adaptive strategy on the feedback gains, several adaptive synchronization criteria were attained for weighted networks Wang, Dai, Dong, Cao, and Sun (2008) and general networks Zhou, Lu, and Lu (2008). A fully decentralized adaptive pinning control scheme for cluster synchronization of undirected networks by introducing local adaptive strategies to both coupling strengths and feedback gains was considered in Delellis, Bernardo, and Turci (2010) and Su, Rong, Wang, and Chen (2010). In order to achieve finite-time synchronization, discontinuous pinning controllers were designed for coupled neural networks Shen and Cao (2011). Without traditional assumptions on control width, a pinning periodically intermittent scheme was designed for pinning control of delayed dynamic networks Cai, Hao, He, and Liu (2011). In order to be effective on the dynamical systems which are subject to instantaneous perturbations at certain instants, pinning control of delayed networks was investigated by a single impulsive controller Zhou, Wu, and Xiang (2011).

(v) Consensus and flocking of multi-agent systems with leaders. Leaders are commonly adopted to help the agents to achieve a desired common velocity and/or to arrive at a desired destination. In switching multi-agent networks, many versions of distributed coordinated control protocols have been investigated towards finding sufficient stability conditions. Based on the artificial potential function method, the coordinated control of multiple autonomous agents with a virtual leader was investigated in Leonard et al. (2001). The consensus problem with an active leader and variable interconnection topology was studied via a neighbor-based state-estimation strategy in Hong, Hu, and Gao (2006). Consensus algorithms with a time-varying virtual leader were investigated for vehicles modeled by single integrator dynamics in Ren (2007). Based on the contraction analysis and multiple Lyapunov functions, the consensus of multi-agent systems with general nonlinear coupling was investigated in Chen, Chen, Xiang, Liu, and Yuan (2009). Distributed coordinated tracking with reduced interaction was investigated via a variable structure method in Cao and Ren (2012). Flocking with a virtual leader was investigated by incorporating information feedback into a few pinning agents Su, Wang, and Lin (2009a).

(vi) Connectivity maintenance. One effective way to achieve coordinated control is to design a protocol which could preserve connectivity of the network during the evolution. In this way, a single informed agent is enough to guide all other agents in the network. Ji and Egerstedt (2007) proposed a hysteresis in adding new links and a special potential function to achieve rendezvous while preserving the network connectivity. Zavlanos, Jadbabaie, and Pappas (2007) used the hysteresis and potential function method to study a flocking problem combined with the network connectivity for double-integrator dynamics to achieve velocity alignment. Based on the bounded potential function method, rendezvous of multi-agents was studied with preserved network connectivity Su, Wang, and Chen (2010). A connectivity-preserving flocking algorithm for multi-agent systems based only on position measurements was proposed in Su, Wang, and Chen (2009). By introducing local adaptive strategies to both the coupling strength and feedback gain, connectivity-preserving consensus and flocking algorithms were proposed for multi-agent nonlinear systems in Su, Chen, Wang, and Lin (2011) and Su, Zhang, Chen, Wang, and Wang (2013), respectively.

In this paper, we review some advances in pinning control of complex networked systems. For a network with fixed topology, we survey synchronization via pinning, which contains some local and global stability conditions for synchronization, virtual control propagation, selective strategies of pinning control, and local adaptive strategies for pinning control. For a network with switching topology, we survey consensus and flocking of multi-agent systems via pinning, which includes second-order consensus of harmonic oscillators without connectivity assumptions, semi-global consensus with input saturation, consensus in a heterogeneous influence network, distributed pinning-controlled flocking without and with preserved network connectivity.

2. Synchronization of complex dynamical networks via pinning

Consider a network consisting of N identical linearly and diffusively coupled nodes of n-dimensional dynamical system, described by

$$\dot{x}_i(t) = f(x_i(t), t) + \sum_{j=1, j\neq i}^{N} c_{ij} a_{ij} \Gamma(x_j(t) - x_i(t)), \quad i = 1, \ldots, N, \tag{1}$$

where $x_i = \left[x_i^1, \ldots, x_i^n\right]^T \in \mathbf{R}^n$ is the state vector of the ith node, $f : \mathbf{R}^n \times [0, +\infty) \to \mathbf{R}^n$ is a continuous map, and c_{ij} denote the coupling strengths between node i and node j, $\Gamma = (\tau_{ij}) \in \mathbf{R}^{n\times n}$ is a matrix linking coupled variables, and if some pairs $(i,j), 1 \leqslant i,j \leqslant n$, with $\tau_{ij} \neq 0$, then it means two coupled nodes are linked through their ith and jth state variables, respectively. If node i can obtain information from node j, then $a_{ij} = 1$; otherwise, $a_{ij} = 0$, and $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$. The coupling matrix $A = (a_{ij}) \in \mathbf{R}^{N\times N}$ represents the coupling configuration of the network. Suppose the network is connected in the sense of having no isolated clusters. Then, the symmetric matrix A is irreducible.

2.1. Stability conditions for synchronization of complex networks

Network (1) is said to realize complete synchronization if

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0$$

for all i and j. The problem of pinning controlled synchronization of network (1) is to directly control a fraction of nodes in the network to achieve

$$\lim_{t\to\infty} \|x_i(t) - \bar{x}(t)\| = 0, \quad i = 1, \ldots, N,$$

where the homogeneous stationary state satisfies

$$\dot{\bar{x}}(t) = f(\bar{x}(t), t). \tag{2}$$

To achieve the goal of control, we apply the pinning control strategy on a small fraction δ $(0 < \delta \ll 1)$ of the nodes in network (1). Suppose that nodes $i_1, i_2, \ldots, i_l$ are selected, where $l = [\delta N]$ stands for the smaller but nearest integer to the real number δN. The controlled network can be described as

$$\begin{aligned}
\dot{x}_{ik} &= f(x_{ik}) + \sum_{j=1, j\neq i_k}^{N} c_{i_k j} a_{i_k j} \Gamma(x_j - x_{i_k}) + u_{i_k}, \quad k = 1, 2, \ldots, l, \\
\dot{x}_{ik} &= f(x_{ik}) + \sum_{j=1, j\neq i_k}^{N} c_{i_k j} a_{i_k j} \Gamma(x_j - x_{i_k}), \quad k = l+1, l+2, \ldots, N
\end{aligned} \tag{3}$$

with

$$u_{i_k} = -c_{i_k i_k} d_{i_k} \Gamma(x_{i_k} - \bar{x}), \quad k = 1, 2, \ldots, l, \tag{4}$$

where the coupling strength $c_{i_k i_k}$ satisfies $c_{i_k i_k} a_{i_k i_k} + \sum_{j=1, j \neq i_k}^{N} c_{i_k j} a_{i_k j} = 0$, and the feedback gain $d_{i_k} > 0$.

Without loss of generality, we rearrange the order of nodes in the network such that the pinned nodes i_k, $k = 1, 2, \ldots, l$, are the first l nodes in the rearranged network.

Define the following two matrices:

$$D = diag(d_1, d_2, \ldots, d_l, 0, \ldots, 0) \in \mathbf{R}^{N \times N},$$

$$D' = diag(c_{11} d_1, c_{22} d_2, \ldots, c_{ll} d_l, 0, \ldots, 0) \in \mathbf{R}^{N \times N}.$$

Substituting (4) into (3), we can rearrange the controlled network (3) and write it by using the Kronecker product as

$$\begin{aligned} \dot{x} &= f(x) - [(G + D') \otimes \Gamma]x + (D' \otimes \Gamma)\overline{X} \\ &= I_N \otimes f(x_i) - [(G + D') \otimes \Gamma]x + (D' \otimes \Gamma)\overline{X}, \end{aligned} \tag{5}$$

where $\overline{X} = [\bar{x}^T, \ldots, \bar{x}^T]^T$, and the elements g_{ij} of the symmetric irreducible matrix $G = (g_{ij}) \in \mathbf{R}^{N \times N}$ are defined as

$$g_{ij} = -c_{ij} a_{ij}. \tag{6}$$

G is positive semi-definite, and $G + D'$ is positive definite with the minimal eigenvalue $\lambda_{\min}(G + D') > 0$.

2.1.1. Global stability conditions

Theorem 1 Su and Wang (2013). *Suppose that Γ is symmetric positive semi-definite. Let T be a matrix such that $f(x) + Tx$ is V-uniformly decreasing for some symmetric positive definite matrix V. The controlled network (5) is globally asymptotically stable about the homogenous state $\bar{x}$ if there exists a positive definite diagonal matrix U such that the matrix*

$$(U \otimes V)[(G + D') \otimes \Gamma + I \otimes T] \tag{7}$$

is positive definite.

Theorem 1 corrects some errors in the Theorem 1 in Li et al. (2004), where it is assumed that U is in a set W_i and (7) is positive semi-definite, and D in (7) is replaced by D'.

Theorem 2 Li et al. (2004). *Assume that $f(x)$ is Lipschitz continuous in x with a Lipshitz constant $L_C^f > 0$. If Γ is symmetric and positive definite, then the controlled dynamical network (5) is globally asymptotically stable about the homogenous state $\bar{x}$, provided that there exists a constant $\alpha = (L_C^f)/(\lambda_{\min}(\Gamma)) > 0$ such that*

$$\lambda_{\min}(G + D') > \alpha, \tag{8}$$

where $\lambda_{\min}(\Gamma)$ and $\lambda_{\min}(G + D')$ are the minimal eigenvalues of matrices Γ and $G + D'$, respectively.

2.1.2. Local stability conditions

Theorem 3 Wang and Chen (2002c). *Consider the controlled network (3). Suppose that there exists a constant $\rho > 0$ such that $[Df(\bar{x}) - \rho]$ is a Hurwitz matrix, $c_{ij} = c, d_i = cd, \Gamma = I_n$. Let $\lambda_1 = \lambda_{\min}(-A + diag(d, \ldots, d, 0, \ldots, 0))$. If*

$$c\lambda_1 \geqslant \rho, \tag{9}$$

then the homogeneous stationary state $\bar{x}$ of the controlled network (3) is locally exponentially stable.

Theorem 4 Li et al. (2004). *Assume that the node $\dot{x}_i = f(x_i)$ is chaotic for all $i = 1, 2, \ldots, N$, with the maximum positive Lyapunov exponent $h_{\max} > 0$. If $c_{ij} = c, d_i = cd, \Gamma = I_n$, then the controlled network (3) is locally asymptotically stable about the homogenous state $\bar{x}$, provided that*

$$c > \frac{h_{\max}}{\lambda_{\min}(-A + diag(d, \ldots, d, 0, \ldots, 0))}. \tag{10}$$

2.2. Virtual control of pinned complex dynamical networks

Let the coupling strength $c_{ij} = c$ and the feedback gain $d_i = cd$, and further set Γ be a 0–1 matrix. One has

$$\begin{aligned} \dot{x}_i &= f(x_i) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j - cd\Gamma(x_i - \bar{x}), \quad i = 1, \ldots, N, \\ \dot{x}_i &= f(x_i) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j, \quad i = l+1,\ l+2, \ldots, N. \end{aligned} \tag{11}$$

Owing to the local error-feedback nature of each pinned node, as the feedback control gain limit $d \to \infty$, the states of the controlled nodes can be pinned to the homogenous target state $\bar{x}$. Hence, the pinning control stability of network (11) can be rewritten in the form with a virtual control as

$$\begin{aligned} x_i &= \bar{x}, \quad i = 1, 2, \ldots, l, \\ \dot{x}_i &= f(x_i) + c\sum_{j=l+1}^{N} \tilde{b}_{ij}\Gamma x_j + \tilde{u}_i \quad i = l+1, l+2, \ldots, N, \end{aligned} \tag{12}$$

where the virtual control laws are taken as

$$\tilde{u}_i = -c\tilde{d}_i(x_i - \bar{x}), \quad i = l+1, l+2, \ldots, N \tag{13}$$

with the virtual control feedback gains $\tilde{d}_i = \sum_{j=1}^{l} a_{ij}$.

Denote $\widetilde{B} = (\tilde{b}_{ij}) \in \mathbf{R}^{(N-l)\times(N-l)}$ as

$$\begin{aligned} \tilde{b}_{ij} &= a_{ij}, \quad j \neq i, \quad j = l+1, \ldots, N, \\ &\qquad i = l+1, l+2, \ldots, N, \\ \tilde{b}_{ii} &= -\sum_{j=1, j\neq i}^{N} a_{ij}, \end{aligned} \tag{14}$$

which is symmetric. Suppose that the nodes $i_1, i_2, \ldots, i_l$ are selected to be under pinning control, and $\widetilde{A} \in \mathbf{R}^{(N-l)\times(N-l)}$ is a minor matrix of A with respect to the pinning control scheme, which is obtained by removing the $i_1, i_2, \ldots, i_l$ row-column pairs from A, we have

$$\widetilde{A} = \widetilde{B} + diag(\tilde{d}_{l+1}, \tilde{d}_{l+2}, \ldots, \tilde{d}_N). \tag{15}$$

Corollary 1 Li et al. (2004). *If $\widetilde{A}$ is irreducible, then the dynamical network (12) with the virtual control (13) is globally (or locally asymptotically) stable about homogenous state $\bar{x}$, provided that $\Gamma = I_m$ and*

$$c > \frac{C}{\lambda_{\min}(-\widetilde{A})} \tag{16}$$

with C being the L_C^f in Theorem 2 (or the $h_{\max}$ in Theorem 4).

Owing to the nonhomogeneous nature of scale-free networks, it is much easier to stabilize a scale-free network by specifically placing the local controllers on the hub nodes with high degrees than by randomly placing these local controllers into the network Wang and Chen (2002c).

2.3. Local adaptive strategies in pinning control for cluster synchronization

Suppose d nonempty subsets (clusters) $\{G_1, \ldots, G_d\}$ is a partition of the index set $\{1, 2, \ldots, N\}$, where $\cup_{l=1}^{d} G_l = \{1, 2, \ldots, N\}$ and $G_l \neq \emptyset$. A network with N nodes is said to realize d-cluster synchronization if

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0$$

for all i and j in the same cluster, and

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| \neq 0$$

for all i and j in different clusters.

The problem of pinning control for d-cluster synchronization is to directly control a small fraction of nodes in network (1) to achieve

$$\lim_{t\to\infty} \sum_{l=1}^{d} \sum_{i\in G_l} \|x_i(t) - \bar{x}_l(t)\| = 0,$$

where $\bar{x}_l(t)$ is the desired state of the lth cluster G_l, which naturally is required to satisfy

$$\dot{\bar{x}}_l(t) = f(\bar{x}_l(t), t), \quad l = 1, \ldots, d, \tag{17}$$

where $\bar{x}_l(t)$ is an equilibrium of the node system. The controlled network is described as follows:

$$\begin{aligned} \dot{x}_i(t) = f(x_i(t), t) &+ \sum_{j=1, j\neq i}^{N} c_{ij} a_{ij} \Gamma(x_j(t) - x_i(t)) \\ &+ h_i c_i \Gamma(\bar{x}_{\hat{i}}(t) - x_i(t)), \quad i = 1, \ldots, N, \end{aligned} \tag{18}$$

where $\hat{i}$ is the subscript of the subset for which $i \in G_{\hat{i}}$. If node i is selected to be pinned, then $h_i = 1$; otherwise, $h_i = 0$.

Unlike other pinning control algorithms, which require global information of the underlying network such as the eigenvalues of the coupling matrix of the whole network or a centralized adaptive control scheme, Su, Rong, et al. (2013) propose a decentralized adaptive pinning control scheme for cluster synchronization of undirected networks using a local adaptive strategy on both coupling strengths and feedback gains. The proposed adaptive strategy for node i is designed as

$$\begin{aligned} &\dot{x}_i(t) = f(x_i(t), t) + \sum_{j=1, j\neq i}^{N} c_{ij}(t) a_{ij} (x_j(t) - x_i(t)) + h_i c_i(t) (\bar{x}_{\hat{i}}(t) - x_i(t)), \\ &\dot{c}_{ij}(t) = h_{ij} a_{ij} k_{ij} (x_i(t) - x_j(t))^T P (x_i(t) - x_j(t)), \\ &\dot{c}_i(t) = h_i k_i (x_i(t) - \bar{x}_{\hat{i}}(t))^T P (x_i(t) - \bar{x}_{\hat{i}}(t)), \end{aligned} \tag{19}$$

where $c_{ij}(0) = c > 0$, $c_i(0) \geqslant 0$, $\Gamma = I_n$ in network (18) and $P = diag\{p_1, \ldots, p_n\}$ is a positive-definite diagonal matrix. The positive constants $k_{ij} = k_{ji}$ and k_i are the weights of the adaptive laws for parameters $c_{ij}(t)$ and $c_i(t)$, respectively. If nodes i and j are in the same cluster, then $h_{ij} = 1$; otherwise, $h_{ij} = 0$. Clearly, the adaptive parameters $c_{ij}(t)$ for node i only contain the state information of its neighbors. It is assumed that for an $N \times N$ symmetric matrix

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1d} \\ A_{21} & A_{22} & \cdots & A_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ A_{d1} & A_{d2} & \cdots & A_{dd} \end{bmatrix}, \tag{20}$$

each block $A_{uv} = (z_{ij}) \in \mathbf{R}^{k_u \times k_v} (u, v = 1, \ldots, d)$ is a zero-row-sum matrix, that is $\sum_{j=1}^{k_v} z_{ij} = 0$, and each block $A_{uu} = (s_{ij}) \in \mathbf{R}^{k_u \times k_u}$ satisfies $s_{ii} = -\sum_{j=1, j\neq i}^{k_u} s_{ij}$ where $s_{ij} = s_{ji} \geqslant 0 (i \neq j)$ and k_u and k_v are the numbers of nodes in the subsets u and v, respectively.

Assumption 1. *For any $x, y \in \mathbf{R}^n$, the continuous map $f : \mathbf{R}^n \times [0, +\infty) \to \mathbf{R}^n$ satisfies*

$$(x - y)^T P\{[f(x, t) - f(y, t)] - \Delta(x - y)\} \leqslant -\omega (x - y)^T (x - y) \tag{21}$$

for a diagonal matrix $\Delta = diag\{\delta_1, \ldots, \delta_n\}$, and a positive constant $\omega > 0$.

Theorem 5 Su, Rong, et al. (2013). *Consider network (18), where each node is steered by the adaptive strategy (19). Suppose that Assumption 1 holds, and at least one node in each cluster is selected to be controlled. Let matrix A be given as in Eq. (20). Then, all clusters asymptotically synchronize to their given heterogeneous stationary states, namely,*

$$\lim_{t\to\infty} \sum_{l=1}^{d} \sum_{i\in G_l} \|x_i(t) - \bar{x}_l(t)\| = 0.$$

Moreover, in order to use only local neighbors' information and design full decentralized coordinated protocols, the local adaptive strategies can also be extended to the consensus Su et al. (2011), Su, Chen, Wang, Wang, and Valeyev (2013), and Hu, Su, and Lam (2013) and flocking Su, Zhang, et al. (2013) of multi-agent systems.

3. Consensus and flocking of multi-agent systems via pinning

Consider a multi-agent system consisting of N agents of n-dimensional dynamical system, described by

$$\dot{x}_i(t) = f(x_i, u_i), \quad i = 1, \ldots, N, \tag{22}$$

where x_i is the state vector of the ith agent, f is a smooth vector field representing its dynamics, u_i is the control input which can only use the states of its neighbors.

The problem of consensus with a virtual leader is to design control input $u_i, i = 1, 2, \ldots, N$, such that

$$\lim_{t\to\infty} \|x_i(t) - x_\gamma(t)\| = 0, \tag{23}$$

for all $i = 1, 2, \ldots, N$, where x_γ is the state of the virtual leader, which satisfies

$$\dot{x}_\gamma = f(x_\gamma). \tag{24}$$

If the state of agent i is composed of its position q_i and velocity p_i, then $x_i = [q_i, p_i]^T$.

The problem of flocking with a virtual leader is to design control input $u_i, i = 1, 2, \ldots, N$, such that each agent can asymptotically approach the velocity of the virtual leader, the distance between any two agents is asymptotically stabilized, and collisions among the agents can be avoided.

For consensus and flocking algorithms, the stability condition contains two main factors, namely, the agent dynamics and the network topology. In the following subsections, we will review consensus and flocking problems with different agent dynamics and different network topologies.

3.1. Second-order consensus of harmonic oscillators without connectivity assumptions

Consider N agents moving in a one-dimensional Euclidean space. The behavior of each agent is described by a harmonic oscillator of the form

$$f(x_i, u_i) = f(q_i, p_i, u_i) = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix} \begin{bmatrix} q_i \\ p_i \end{bmatrix} + \begin{bmatrix} 0 \\ u_i \end{bmatrix}, \tag{25}$$

where ω is the frequency of the oscillator.

The dynamic of the leader satisfies

$$f(x_\gamma) = f(q_\gamma, p_\gamma) = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix} \begin{bmatrix} q_\gamma \\ p_\gamma \end{bmatrix}. \tag{26}$$

Each agent has a limited communication capability which allows it to communicate only with agents within its neighborhood. The neighboring agents of agent i at time t is denoted as:

$$\mathcal{N}_i(t) = \{j : \|q_i - q_j\| < r,\ j = 1, 2, \ldots, N,\ j \neq i\},$$

where $\|\cdot\|$ is the Euclidean norm.

Let the control input for agent i be given by

$$u_i = -\sum_{j\in\mathcal{N}_i(t)} a_{ij}(q)(p_i - p_j), \quad i = 1, 2, \ldots, N, \tag{27}$$

where $A(q) = (a_{ij}(q))_{N\times N}$ is the adjacent matrix. Denote the position and velocity of the center of mass (COM) of all agents in the group as

$$\bar{q} = \frac{\sum_{i=1}^{N} q_i}{N}, \quad \bar{p} = \frac{\sum_{i=1}^{N} p_i}{N}.$$

Theorem 6 Su, Wang, and Lin (2009b). *Consider a system of N mobile agents with dynamics (25), each being steered by the control input (27). Then,*

$$q_i(t) \to \bar{q}(0)\cos(\omega t) + \frac{1}{\omega}\bar{p}(0)\sin(\omega t)$$

and

$$p_i(t) \to -\omega\bar{q}(0)\sin(\omega t)\bar{q}(0) + \bar{p}(0)\cos(\omega t),$$

as $t \to \infty$, where $\bar{q}(0)$ and $\bar{p}(0)$ are respectively the initial position and velocity of the COM of the group.

In the case with a leader, the control input for agent i is given by

$$u_i = -\sum_{j\in\mathcal{N}_i(t)} a_{ij}(q)(p_i - p_j) - a_{i\gamma}(q)(p_i - p_\gamma), \quad i = 1, 2, \ldots, N. \tag{28}$$

Theorem 7 Su et al. (2009b). *Consider a system of N mobile agents with dynamics (25) and a leader with dynamic (26). Let each follower be steered by the control input (28). Then,*

$$q_i(t) \to q_\gamma(0)\cos(\omega t) + \frac{1}{\omega}p_\gamma(0)\sin(\omega t)$$

and

$$p_i(t) \to -\omega q_\gamma(0)\sin(\omega t) + p_\gamma(0)\cos(\omega t),$$

as $t \to \infty$, where $q_\gamma(0)$ and $p_\gamma(0)$ are respectively the initial position and velocity of the leader.

Unlike many existing algorithms, Theorems 6 and 7 show that the coupled harmonic oscillators can be synchronized even without any network connectivity assumption. This is because there exist an infinite sequence of contiguous, nonempty and uniformly bounded time-intervals $[t_j, t_{j+1})$, $j = 0, 1, 2, \ldots$, such that across each time interval each pair of agents in the group must naturally collide, that is, they exchange velocity information with each other when the relative distances between them are within the influencing/sensing radius r across each time interval.

3.2. Semi-global consensus with input saturation

In this subsection, we consider a group of N agents with general linear dynamics. The motion of each agent is described by

$$\begin{aligned} \dot{x}_i &= f(x_i, u_i) = Ax_i + B\sigma(u_i), \\ y_i &= Cx_i, \quad i = 1, 2, \ldots, N, \end{aligned} \tag{29}$$

where $x_i \in \mathbf{R}^n$ is the state of agent $i, y_i \in \mathbf{R}^p$ is the measurement output of agent $i, u_i \in \mathbf{R}^m$ is the control input acting on agent i, and $\sigma : \mathbf{R}^m \to \mathbf{R}^m$ is a saturation function defined as $\sigma(u_i) = [\mathbf{sat}(u_{i1})\ \mathbf{sat}(u_{i2})\ \cdots\ \mathbf{sat}(u_{im})]^T$, $\mathbf{sat}(u_{ij}) = \mathbf{sgn}(u_{ij})\min\{|u_{ij}|, \varpi\}$, for some constant $\varpi > 0$. The dynamics of the leader, labeled as $N+1$, is described by

$$\begin{aligned} \dot{x}_{N+1} &= Ax_{N+1}, \\ y_{N+1} &= Cx_{N+1}. \end{aligned} \tag{30}$$

The problem of semi-global (observer-based) leader-following consensus for the agents and leader described above is the following: For any *a priori* given bounded set $\mathbf{X} \subset \mathbf{R}^n$, construct a control law u_i for each agent i, which use only local information (measurement outputs) from neighbor agents, such that

$$\lim_{t\to\infty}\|x_i(t) - x_{N+1}(t)\| = 0, \quad i = 1, 2, \ldots, N,$$

as long as $x_i(0) \in \mathbf{X}$ for all $i = 1, 2, \ldots, N, N+1$.

Assumption 2. *The pair (A, B) is asymptotically null controllable with bounded controls, that is,*

(1) (A, B) is stabilizable.
(2) All the eigenvalues of A are in the closed left-half s-plane.

Assumption 3. *The pair (A, C) is detectable.*

The low gain feedback design for the multi-agent system (29) is carried out in two steps.

Step 1. Solve the parametric algebraic Riccati equation (ARE)

$$A^{\mathrm{T}}P(\varepsilon) + P(\varepsilon)A - 2\gamma P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon) + \varepsilon I = 0, \quad \varepsilon \in (0, 1], \tag{31}$$

where $\gamma \leqslant \min\{\lambda_1(L_s + H)\}$ is a positive constant, and L_s is the Laplace matrix of possible spanning trees consisting of the N agents.

Step 2. Construct a linear feedback law for agent i as

$$\begin{aligned} u_i = &-B^{\mathrm{T}}P(\varepsilon)\sum_{j=1}^{N} a_{ij}(t)(x_i - x_j) \\ &- B^{\mathrm{T}}P(\varepsilon)h_i(t)(x_i - x_{N+1}), \quad i = 1, 2, \ldots, N. \end{aligned} \tag{32}$$

Theorem 8 Su, Chen, Lam, and Lin (2013). *Consider a multi-agent system of N agents with general linear dynamics (29) and a leader with dynamics (30). Suppose that Assumption 2 holds and the network is connected. Then, the agent control inputs u_i as given by (32) achieve semi-global consensus. That is, for any a priori given bounded set $\mathbf{X} \subset \mathbf{R}^n$, there is an $\varepsilon^* > 0$ such that, for each given $\varepsilon \in (0, \varepsilon^*]$,*

$$\lim_{t\to\infty}\|x_i(t) - x_{N+1}(t)\| = 0, \quad i = 1, 2, \ldots, N,$$

as long as $x_i(0) \in \mathbf{X}$ for all $i = 1, 2, \ldots, N, N+1$.

The low gain output feedback designed for the multi-agent system (29) is carried out in two steps.

Step 1. Solve the parametric ARE

$$A^TP(\varepsilon) + P(\varepsilon)A - \gamma P(\varepsilon)BB^TP(\varepsilon) + \varepsilon I = 0, \quad \varepsilon \in (0, 1]. \tag{33}$$

Step 2. Construct a linear output feedback law for agent i as

$$\begin{aligned} \dot{\hat{x}}_i &= A\hat{x}_i - F(y_i - C\hat{x}_i) \\ &\quad - BB^TP(\varepsilon)\left(\sum_{j=1}^{N} a_{ij}(t)(\hat{x}_i - \hat{x}_j) + h_i(t)(\hat{x}_i - \hat{x}_{N+1})\right), \\ u_i &= -B^TP(\varepsilon)\left(\sum_{j=1}^{N} a_{ij}(t)(\hat{x}_i - \hat{x}_j) + h_i(t)(\hat{x}_i - \hat{x}_{N+1})\right), \\ &\quad i = 1, 2, \ldots, N, \\ \dot{\hat{x}}_{N+1} &= A\hat{x}_{N+1} - F(y_{N+1} - C\hat{x}_{N+1}), \end{aligned} \tag{34}$$

where $\hat{x}_i \in \mathbf{R}^n$ is the protocol state of agent i, and $F \in \mathbf{R}^{n\times p}$ is the feedback gain matrix, which is chosen such that $(A+FC)$ is asymptotically stable. The existence of such an F is guaranteed by Assumption 3.

Theorem 9 Su, Chen, Wang, and Lam (2014). *Consider a multi-agent system of N agents with general linear dynamics (29) and a leader with dynamics (30). Suppose that Assumptions 2 and 3 hold and the network is connected. The control inputs u_i for the agent (34) achieve semi-global consensus. That is, for any given bounded set $\mathbf{X} \subset \mathbf{R}^n$, there is an $\varepsilon^* > 0$ such that, for each $\varepsilon \in (0, \varepsilon^*]$,*

$$\lim_{t\to\infty} \|x_i(t) - x_{N+1}(t)\| = 0, \quad i = 1,2,\ldots,N,$$

as long as $x_i(0) \in \mathbf{X}$ for all $i = 1,2,\ldots,N,N+1$.

Furthermore, both semi-global state feedback consensus Su, Chen, Lam, et al. (2013) and output feedback consensus Su et al. (2014) with input saturation are investigated in a more practical scene, that is, the agents get in touch, directly or indirectly, with the leader from time to time.

3.3. Consensus in a heterogeneous influence network

Yang, Cao, Wang, and Li (2006) investigated consensus in a heterogeneous influence network. Different from the classical Vicsek model Vicsek, Cziróok, Ben-Jacob, Cohen, and Shochet (1995), the influencing capability of each agent is represented by its influencing radius, which is randomly chosen according to a power law distribution with a scaling exponent between 2 and ∞. As the value of the scaling exponent decreases, the radius distribution becomes more heterogeneous and the network becomes much easier to achieve direction consensus among agents due to the leading roles played by a few hub agents. Furthermore, almost all agents will finally move in the same desired direction in a strong heterogeneous influence network, if and only if a small fraction of hub agents can be controlled to move in the desired direction. These results also reflect the ‘robust yet fragile’ feature of a heterogeneous influence network.

3.4. Flocking with a virtual leader

Different from consensus algorithm, the spirit of flocking algorithm is inspired by the pioneering model of Reynolds (1987). Consider N agents moving in an n dimensional Euclidean space. The motion of each agent is described by two integrators as

$$\begin{cases} \dot{q}_i = p_i, \\ \dot{p}_i = u_i, \quad i = 1,2,\ldots,N. \end{cases} \tag{35}$$

The virtual leader is with the following model of motion,

$$\begin{cases} \dot{q}_\gamma = p_\gamma, \\ \dot{p}_\gamma = f_\gamma(q_\gamma, p_\gamma), \end{cases} \tag{36}$$

with $(q_\gamma(0), p_\gamma(0)) = (q_d, p_d)$.

The neighboring set of agents is the same as that in Section 3.1. The control protocol in the second flocking algorithm of Olfati-Saber (2006) is given by,

$$\begin{aligned} u_i = &-\sum_{j\in\mathcal{N}_i(t)} \nabla_{q_i}\psi_\alpha(\|q_j - q_i\|_\sigma) \\ &+ \sum_{j\in\mathcal{N}_i(t)} a_{ij}(q)(p_j - p_i) - c_1(q_i - q_\gamma) - c_2(p_i - p_\gamma), \quad c_1, c_2 > 0, \end{aligned} \tag{37}$$

where the function $\psi_\alpha : \mathbf{R}_+ \to \mathbf{R}_+$ is a nonnegative smooth pairwise potential function of the distance $\|q_{ij}\|_\sigma$ between agents i and j, such that ψ_α reaches its maximum as $\|q_{ij}\|_\sigma \to 0$, attains its unique minimum when agents i and j are located at a desired distance $\|d\|_\sigma$, and is constant for $\|q_{ij}\|_\sigma \geqslant \|r\|_\sigma$.

In the flocking algorithm (37), it is assumed that each agent is an informed agent who has the information about the virtual leader (*i.e.*, its position and velocity). Here, it is assumed that only some, but not all, of the agents are informed agents who are given the information about the virtual leader. Consequently, the control input for agent i, (37), is modified as

$$\begin{aligned} u_i = &-\sum_{j\in\mathcal{N}_i(t)} \nabla_{q_i}\psi_\alpha(\|q_j - q_i\|_\sigma) + \sum_{j\in\mathcal{N}_i(t)} a_{ij}(q)(p_j - p_i) \\ &- h_i[c_1(q_i - q_\gamma) + c_2(p_i - p_\gamma)], \quad c_1, c_2 > 0, \end{aligned} \tag{38}$$

where $h_i = 1$ if agent i is informed and $h_i = 0$ otherwise.

Denote the union of all neighboring graphs across a nonempty finite time interval $[t_i, t_{i+1}), t_{i+1} > t_i$ as $\widehat{\mathcal{G}}(t_i, t_{i+1})$, whose edges are the union of the edges of those neighboring graphs. For an uninformed agent, if there is a path between this agent and one informed agent in the union $\widehat{\mathcal{G}}(t_i, t_{i+1})$, then one can say that there exists a joint path between the uninformed agent and the informed agent across the finite time interval $[t_i, t_{i+1})$. An uninformed agent is called a type I uninformed agent if there exists an infinite sequence of contiguous, nonempty and uniformly bounded time-intervals $[t_i, t_{i+1}), i = 0,1,2,\ldots$, such that across each time interval there exists a joint path between this agent and one informed agent. Otherwise, it is called a type II uninformed agent.

Define the sum of the total artificial potential energy and the total relative kinetic energy between all agents and the virtual leader as follows,

$$Q(q,p) = \frac{1}{2}\sum_{i=1}^{N}\left[U_i(q) + (p_i - p_\gamma)^{\mathrm{T}}(p_i - p_\gamma)\right], \tag{39}$$

where

$$U_i(q) = \sum_{j=1, j\neq i}^{N} \psi_\alpha(\|q_i - q_j\|_\sigma) + h_i c_1 (q_i - q_\gamma)^{\mathrm{T}}(q_i - q_\gamma). \tag{40}$$

Theorem 10 Su et al. (2009a). *Consider a system of N mobile agents, each with dynamics (35) and steered by the control protocol (38). Suppose that the initial energy $Q_0 := Q(q(0), p(0))$ is finite. Then the following statements hold:*

(*i*) *The distance between each informed agent and the virtual leader is not greater than $\sqrt{2Q_0/c_1}$ for all $t \geqslant 0$.*
(*ii*) *The velocities of all informed agents and type I uninformed agents approach the desired velocity p_γ asymptotically.*
(*iii*) *If the initial energy Q_0 of the group is less than $(\bar{k}+1)c^*,\ c^* = \psi_\alpha(0)$, for some $\bar{k} \in \mathbf{Z}_+$, then at most $\bar{k}$ distinct pairs of agents could possibly collide ($\bar{k} = 0$ guarantees a collision free motion).*

A fraction δ of the N agents are randomly selected as the informed agents. Fig. 1 shows the relationship between the fraction η of the agents that eventually move with the desired velocity and the fraction δ of the informed agents with $N = 100, 300, 500$ and 1000, respectively. All estimates are the results of averaging over 50 realizations. Obviously, for any given group size N, the fraction η of agents that move with the desired velocity is an increasing function of the fraction δ of informed agents. Furthermore, the larger the group, the smaller the fraction δ of informed agents is needed to guide the group with a given fraction η. For example, in order for 80% of the agents to move with the same desired velocity, about 27% of the agents should be informed agents when the group size is $N = 100$, but only about 10% of the agents need to

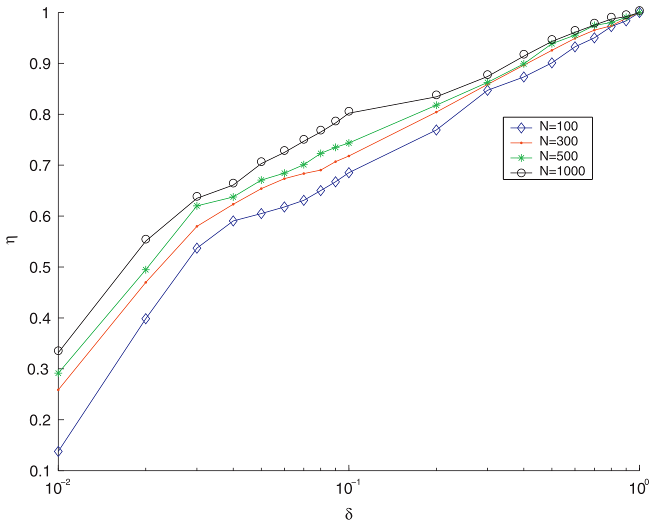

Fig. 1. Su et al. (2009a) fraction of agents with the desired velocity as a function of informed agents. All estimates are the results of averaging over 50 realizations.

be the informed agents when the group has $N = 1000$ agents. Thus, for sufficiently large groups, only a very small fraction of informed agents will guide most agents in the group.

In reality, there are many missions which include multiple goals seeking. Therefore, the problem of flocking with multiple virtual leaders is also an interesting and important investigation, which is studied in Su, Wang, and Yang (2008).

3.5. Flocking with preserved network connectivity

Subjected to limited sensing and communication capabilities of agents, the interaction topology among agents may change over time. A basic assumption made in most previous works on stability analysis of collective dynamics is that the underlying topology can remain connected enough frequently during the motion evolution. However, for a given set of initial states and parameters, it is very difficult or even impossible to satisfy and verify this assumption in practice. In fact, the connectivity of the initial network generally cannot guarantee the connectivity of the whole network throughout a long process of motion. Our objective is to design control inputs so that all agents attain stable flocking motion while preserving network connectivity.

A flocking algorithm using only position measurements was proposed in Su, Wang, and Chen (2009), which is described as follows:

$$\begin{cases} u_i = -\sum_{j\in N_i(t)} (\nabla_{q_i}\psi(\|q_{ij}\|) - w_{ij}(y_i - y_j)), \\ y_i = PT\hat{x}_i + P\sum_{j\in N_i(t)} w_{ij}q_{ij}, \\ \dot{\hat{x}}_i = T\hat{x}_i + \sum_{j\in N_i(t)} w_{ij}q_{ij}, \end{cases} \tag{41}$$

where $\hat{x}_i, y_i \in \mathrm{R}^n$, $y = [y_1^T, \ldots, y_N^T]^T$, $q_{ij} = q_i - q_j$, $T \in \mathrm{R}^{n\times n}$ is a Hurwitz matrix, $P \in \mathrm{R}^{n\times n}$ is a symmetric positive-definite matrix such that $T^TP + PT = -Q$ is a symmetric positive-definite matrix, the constant $w_{ij} = w_{ji} > 0$ for all $i,j \in V$, ∇ is the gradient operator, $\psi(\cdot)$ is a potential function, $N_i(t)$ is the neighborhood of agent i at time t, and

$$N_i(t) = \{j|\sigma(i,j)[t] = 1,\ j \neq i,\ j = 1,\ldots,N\}. \tag{42}$$

The nonnegative potential $\psi(\|q_{ij}\|)$ is a function of the distance $\|q_{ij}\|$ between agent i and agent j, which is differentiable for $\|q_{ij}\| \in (0, r)$, satisfying

(i) $\psi(\|q_{ij}\|) \to \infty$ as $\|q_{ij}\| \to 0$ or $\|q_{ij}\| \to r$;
(ii) $\psi(\|q_{ij}\|)$ attains its unique minimum when $\|q_{ij}\|$ equals a desired distance.

The energy function for the system is defined as follows:

$$W = \frac{1}{2}\sum_{i=1}^{N}\left(\sum_{j\in N_i(t)} \psi(\|q_{ij}\|) + p_i^Tp_i\right) + \frac{1}{2}\dot{\hat{x}}^T(I_N \otimes P)\dot{\hat{x}}. \tag{43}$$

Theorem 11 Su, Wang, and Chen (2009). *Consider a system of N mobile agents with dynamics (35), each being steered by protocol (41). Suppose that the initial network $G(0)$ is connected and the initial energy $W_0 = W(\widehat{q}(0), p(0), \hat{x}(0))$ is finite. Then, the following hold:*

(i) $G(t)$ will remain to be connected for all $t \geqslant 0$;
(ii) all agents asymptotically move with the same velocity;
(iii) almost every final configuration locally minimizes each agent's global potential $\sum_{j\in N_i(t)} \nabla_{q_i}\psi(\|q_{ij}\|)$;
(iv) collisions between the agents are avoided.

In the case with a virtual leader, we assume that only one agent is being informed about the virtual leader. The control input for agent i is designed as

$$\begin{cases} u_i = -\sum_{j\in N_i(t)} (\nabla_{q_i}\psi(\|q_{ij}\|) - w_{ij}(y_i - y_j)) - h_iy_i, \\ y_i = PT\hat{x}_i + P\sum_{j\in N_i(t)} w_{ij}q_{ij} + Ph_i(q_i - q_\gamma), \\ \dot{\hat{x}}_i = T\hat{x}_i + \sum_{j\in N_i(t)} w_{ij}q_{ij} + h_i(q_i - q_\gamma), \end{cases} \tag{44}$$

where q_γ is the position of the virtual leader. If agent i is the informed agent, then $h_i = 1$; otherwise, $h_i = 0$. Without loss of generality, we assume that the first agent is the informed agent, that is, $h_i = 1$ for $i = 1$ and $h_i = 0$ for all the others.

The energy function for the system is defined as follows:

$$U = \frac{1}{2}\sum_{i=1}^{N}\left(\sum_{j\in N_i(t)} \psi(\|q_{ij}\|) + (p_i - p_\gamma)^T(p_i - p_\gamma)\right) + \frac{1}{2}\dot{\hat{x}}^T(I_N \otimes P)\dot{\hat{x}}. \tag{45}$$

Theorem 12 Su, Wang, and Chen (2009). *Consider a system of N mobile agents with dynamics (35), each being steered by protocol (44). Suppose that the initial network $G(0)$ is connected and the initial energy $U_0 = U\left(\widehat{q}(0), p(0), \hat{x}(0)\right)$ is finite. Then, the following hold:*

(i) $G(t)$ will remain to be connected for all $t \geqslant 0$;
(ii) all agents asymptotically move with the desired velocity p_γ;
(iii) almost every final configuration locally minimizes each agent's global potential $\sum_{j\in N_i(t)} \nabla_{q_i}\psi(\|q_{ij}\|)$;
(vi) collisions between the agents are avoided.

It should be noted, however, that the potential functions constructed above tend to infinity as the distance between two already connected agents tends to the sensing radius, which is a bit impractical. Therefore, we have also constructed a class of bounded potential functions to guarantee the existing links not to be lost Su, Wang, et al. (2010).

4. Conclusions

We have surveyed some recent advances in pinning control of complex networked systems. However, some challenging theoretical problems need to be investigated, including controllability and dynamical mechanisms. Moreover, we still need a deep understanding of the relationship between the effectiveness of pinning control and the structure properties of a network. Finally, note that we have been focused on pinning control to making a dynamical networked system have a desired coordinated behavior. Liu, Slotine, and Barabási (2011) investigated the general controllability of a complex directed network, identifying the set of driver nodes that can guide the system's entire dynamics, and therefore opening new avenues for the control of large-scale complex systems..

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